Changes in the attitudes and behaviour of primary pupils while moving from traditional class teaching to group work.

Panagakos, Ioannis S

CHANGES IN THE ATTITUDES AND BEHAVIOUR OF PRIMARY PUPILS WHILE MOVING FROM TRADITIONAL CLASS TEACHING TO GROUP WORK

Ioannis S. Panagakos

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ABSTRACT

This research is in response to an interest in groupwork practices. Concerns among colleagues in Greece, about the lack of collaboration within Greek primary classrooms prompted this investigation. The purpose of this research was to explore possible changes in students' attitudes and behaviour when moving from traditional class teaching to group work.

This research is based on the theoretical background of social constructivism (e.g. Vygotsky, Cobb). In exploring effects of group work it builds on the work of, for example, Johnson & Johnson, Slavin, Webb, Galton & Williamson.

The investigation was carried out in the context of word problem solving in two grade-four classes of a Greek primary school in England. Data was gathered and analysed mainly using interpretative methods, but some quantitative methods were incorporated. The methodological approach consisted of case study combined with elements of teaching experiment and action research. After a pilot study, two linked case studies were carried out. Pre-tests and post-tests were administered, pre-interviews and post-interviews were conducted, pre-questionnaires and post-questionnaires were completed, and whole class and group conversations were tape-recorded.

The analysis of the data collected suggests that both case studies were successful in establishing a collaborative environment. In both cases, the students changed their attitudes towards group work and problem solving, and some changes regarding their learning behaviour were also observed. The style of teaching appeared to be important in facilitating classroom debate, both in group and whole class discussion.

The unique contribution of this study is that it looks closely at the processes involved during a change from a very traditional class (teaching) to a group working scheme carried out in real classrooms with real students, and reports the problems that arose.

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TABLE OF CONTENTS

CHAPTER 2 THE DESIGN OF THE RESEARCH

CHAPTER 3 TEACHING EXPERIMENT-1 (TE-1)

CHAPTER 4 TEACHING EXPERIMENT-2 (TE-2)

LIST OF TABLES

LIST OF FIGURES

LIST OF APPENDICES

CHAPTER 1 THE BACKGROUND TO THE STUDY, CONTEXT, THEORY AND THE PREVIOUS RESEARCH

1.1 Introduction: the context of the research

In this introductory section I describe briefly the Greek educational system to which my research refers and explain its aim. First I provide a brief introduction to the Greek educational system, with emphasis on the primary school, and describe the context of primary education in Greece. Then I refer to the reasons and conditions that influenced the design of this research. That is, I report on my teaching experiences concerning the lack of collaboration in Greek primary classrooms and the students' difficulties in solving word problems.

1.1.1 A brief introduction to the Greek educational system

The purpose of this section is to convey to the reader the general background of the educational system in Greece, in terms of its structure and organisation.

In Greece, there are three educational levels: primary education (nursery and primary schools), secondary, (gymnasia and lycea) and tertiary which encompasses university-level education (AEI) and non-university (TEI). Compulsory education is spread over 9 years, 6 years at primary and 3 years at gymnasium (secondary school).

Responsibility for the administration and operational supervision of primary and secondary schools lies with senior staff of the Education Directorates or Offices. Teachers and administrative employees serving in the directorates or offices are under the control of these staff. At a similar level the responsibility for issuing guidance on teaching matters, rests with the Schools Adviser whose duty it is to assess the performance of teachers and to arrange for their further training, as well as to encourage educational research.

The role of the primary school is to promote the multi-faceted mental and physical development of pupils in the context of the wider purpose set by primary and secondary education (Law 1566/1985). Attendance at primary school is obligatory and the enrolment of children at primary school is subject to their reaching the age of 5 1/2 years by 1 October of the first year of attendance. All students commence their primary education at the same point in the year.

Pupil assessment for grades 1-3 is conducted orally according to the following grades: Excellent (A), Very Good (B), Good (C). For grades 4-6 the assessment is conducted both orally and in writing using a ten grade scale. Progression to the next class is automatic at the end of each school year. Only when a pupil has for a good reason been absent for more than half the academic year is he or she obliged to sit a progress examination. Failure in such an examination may result in the pupil having to repeat the year. On satisfactory completion of their primary school studies pupils are given a certificate which they use for enrolment at a state gymnasium.

The Pedagogical Institute (PI) operates as an independent state service under the direct jurisdiction of the Minister of National Education and Religion. PI is responsible for the structural and administrative aspects of the primary and secondary education, comprising:

(1) Research into, and study of primary and secondary education issues.
(2) The drafting and submission of proposals on the orientation, planning and programming of educational policy in order to achieve the aims of primary and secondary education in conjunction with the country's economic, social and cultural development programme.
(3) Monitoring of developments in educational technology, study of ways of utilising this technology in education, and monitoring of the results of its application.
(4) Planning and supervision of in-service teacher training programmes.

1.1.2 The context of primary education in Greece

There are serious concerns among mathematicians and educationists regarding the teaching of mathematics in primary school. The back-to-basics reform movement was unsuccessful in bringing about change in the classroom. The results of empirical researches (Pottari, 1989, Hatzigeorgiou, 1990, Tresou - Milona 1991, Kothali and Georgakakos 1992), conducted with students both from primary and gymnasium, confirmed: students' failure in solving word problems; in performing the four basic operations; in executing addition and subtraction with carrying; in ordering decimal numbers. The Pedagogical Institute in an effort to remedy the situation revised in 1993 the mathematical curriculum for the three upper grades in primary school. This revision could be considered as a response to the international problem solving movement.

The classroom teacher is required by law to implement the established mathematics curriculum and to follow through with the objectives which were decided without his cooperation. The students are regarded as passive receivers of the curriculum designers' objectives and they are expected to learn by heart a plethora of mathematical content which is not related to their everyday experiences and/or their individual requirements.

The mathematics curriculum in Greek primary school is linear, binding, closed and tends to favour activities which are structured and well organised. In effect, the mathematics curriculum tends to be categorical, inactive and redundant. An interesting curriculum would be the best ally to the teacher, but such a curriculum is non-existent. Although the National Curriculum follows the reform movements (with some delay), I think in its basic structure it remains traditional (i.e. use of one textbook).

The lack of research papers on how the Greek mathematics curriculum is implemented in primary classrooms precludes me from having a global picture and making general claims. However I will draw some unconfirmed conclusions based on my own teaching experiences, in four different schools in Greece, and on conversations with colleagues.

Despite the fact that the curriculum has certain limitations, it is not completely inflexible and allows for pedagogical innovations. The teacher has some freedom in terms of what to teach. This allows for new ideas and methods and could inspire the currently inactive interest of the children.

My opinion is that the results of the mathematics curriculum as it is implemented today are mostly superficial, as the students tend to learn the mathematical content in a mechanical way. They may state formulae and definitions without understanding them. Generally, they know, for example, how to perform the basic operations but they are not able to explain when and why they should apply each of them. Emphasis is given to the correct answer rather than to the process for arriving at it. Overall success in such a learning system means to provide the correct answer to the problem posed by the book or the teacher.

Despite the many changes and revisions in the National Curriculum in the Greek primary school, it is my impression that very little has changed in practice. Traditionally, the teaching of mathematics has been a transmission of knowledge from the teacher to the students. In other words, teachers adopt a 'bank' view of education, instilling ready-made knowledge in the 'empty minds' of the students.

1.1.3 Aim of the research

My research resulted from an interest which both questioned the lack of collaboration going on in the Greek primary classrooms and the students' difficulties in solving word problems.

I have observed in the Greek primary schools an overriding emphasis on competition and individual work, and a lack of opportunity to work as a group. Where I have found colleagues using group work, I tried to find out the rationales they used for grouping the children in their classes. From discussions, it seems that pragmatic strands of organisation tend to dominate where the management of personalities and creation of groups that were socially cohesive take precedence over groupings for other educational reason. All my colleagues talked about the lack of collaboration within their classrooms, in particular within groups of more than two individuals.

My teaching experience has also indicated that a large number of students have difficulty in solving word problems. They engage easily in computational skills but they appear to be uninterested in solving word problems which require the four basic operations singly or in combination. I have noticed that only a small percentage of the students I have taught accept word problems and try to interpret them in order to find a solution. The majority of students tend to wait for the teacher's or the other pupils' helpful response, while others perform calculations, picking the numbers given in the word problem at random.

Therefore the aim of my research was to investigate possible changes in the attitudes and behaviour of primary pupils while moving from traditional class teaching to group work, in the context of word problem solving.

1.2 Review of literature on collaborative group work

In this section I review the literature on group work. There is clear evidence that group work under certain conditions can facilitate achievement and affective gains. However, there is little research attention on what happens during group work and which factors are important for achieving these goals. It is therefore in this area that I wish to focus my work.

1.2.1 What is collaborative group work?

Some educators believe that small group instruction can be a useful strategy in the learning of mathematics. That is, they argue that use of small groups can offer students more opportunities for interaction and can lead to more meaningful assignments. In fact 'there is a substantial body of theory and research to justify cooperative learning' (Dunne and Bennett, 1990, p. 3). However before discussing the rationale in section 1.2.2 the question arises as to what is cooperative learning or group work?

Slavin (1989a) suggests that 'cooperative learning is a form of classroom organisation in which students work in small groups to help one another learn academic material' (p. 129).

Artzt and Newman (1990) state that 'cooperative learning involves a small group of learners, who work together as a team to solve a problem, complete a task, or accomplish a common goal' (p. 2).

Galton and Williamson (1992, p. 10) have categorised group work in primary classrooms into four types: seating groups, working groups, cooperative group and collaborative group. Their classification (Figure 1.1) is based on the nature of the 'task demand' and the 'intended outcome' of children's work.

Before I continue with the cooperative method employed in this research I want to present a distinction, made by Galton and Williamson (1992) between cooperation and collaboration. This distinction is important as it does affect the type of task that may be assigned to a group. Galton and Williamson (1992) define cooperative group work as 'the situation where pupils work on the same task but each have individual assignment which eventually are put together to form a joint outcome' (p. 10). Collaborative group work 'involves all children contributing to a single outcome and often involves problem-solving activities, particularly in cases where the group has to debate a social or moral issue and

Figure 1.1 Galton and Williamson's (1992) classification of grouping arrangements in the primary class

Type	Task demand	Intended outcome
1 Seating groups	Each pupil has a separate task	Different outcomes: each pupil completes a different assignment
2 Working groups	Each pupil has the same task	Same outcome: each pupil completes the same assignment independently
3 Co-operative group	Each pupil has separate but related task	Joint outcome: each pupil has a different assignment
4 Collaborative group	Each pupil has same task	Joint outcome: all pupils share same assignment

produce an agreed solution or recommendations' (p. 10). Moreover, this definition implies that group members must realise that they are part of a team and that they have a common goal. Success or failure of the group will be shared by all group members. To accomplish the group's goal, students need to talk with one another and provide help, support and encouragement.

Pollard (1997) also discussing group work has identified four main types of groups: (1) task allocation, (2) teaching groups, (3) seating groups, and (4) collaborative groups (p. 210).

(1) Task allocation: When the teacher allocates a task, he/she may have a group of children in mind, though they may not sit together. In other words this form of group may exist only within the teacher's mind.

(2) Teaching groups: Children who are of the same age, doing the same task, at the same time may be grouped together for instructional purposes.

(3) Seating groups: A number of children (4-6) sit together at a table. This form of grouping is flexible as it allows children to work individually and to communicate when it is necessary.

(4) Collaborative groups: Is the most developed form of group work. In this type, all children share a common aim, they work together and they contribute to a joint outcome.

In the past 15 years there have been several models used in the United States for organising classrooms so that students can learn academic material from one another in small groups (Slavin, 1989a). However these collaborative methods vary considerably. According to Slavin (1985) the most widely researched collaborative methods are: (1) student team learning, (2) jigsaw, (3) learning together and (4) group investigation. Some of these are rigidly structured and authoritarian in nature, and may be unsuitable for the teacher who is concerned with student-centred learning. Below I describe briefly the characteristics of these widely used collaborative learning methods.

(1) Student team learning (Developed by Robert Slavin, Nancy Maden, David DeVries and their associates): Is a set of collaborative learning methods which consists of Student Teams - Achievement Divisions (STAD), Teams - Games - Tournament (TGT), Team Assisted Individualization (TAI) and Cooperative Integrated Reading and Composition (CIRC). Three basic elements of all student team methods are team rewards, individual accountability and equal opportunity for success. Teams may get certificates or other rewards if they meet certain criteria. Individual accountability means that the team's success depends on the individual learning of all its members. Equal opportunities for success are provided to students to improve their own earlier performance.

(2) Jigsaw (Designed by Elliot Aronson and his colleagues): Jigsaw was one of the first collaborative learning methods. The students work in small groups of mixed ability, the material to be learnt is broken into sections and each group

member studies one section of the topic that the whole class is studying. Afterwards the students meet with members of other groups who have been assigned the same section, to exchange information and master the material of that section. Then, the students return to their original group and teach their group-mates what they have learned. At the end they are submitted to individual quizzes or tests based on the entire unit. (A modification of jigsaw, is the jigsaw II method developed by Slavin, R.).

(3) Group investigation (Developed by Yael Sharal and Rachel Hertz - Lazarowitz): This method is the most complex of all the collaborative learning methods. The students work in groups of their choice and take substantial responsibility for choosing topics, organising the work themselves and deciding how to communicate what they have learned to their classmates. Each group works on a different topic and the class appears to be 'a group of groups'.

(4) Learning together (Developed by David Johnson and Roger Johnson): The students work in small, mixed ability groups and on the same task. There are two main features which distinguish this collaborative method from others: (i) students receive training in social and interpersonal skills (i.e. to provide support and feedback to each other) and (ii) there is only one product or outcome from each group. 'Evidence suggests that this approach can enhance cognitive processing, peer support and encouragement, self-esteem and time on task' (Askew and Wiliam, 1995, p. 36).

Although these methods have been widely reviewed and researched, the list is neither exclusive nor exhaustive. Adaptations to these methods can be made. However, the rigidity of some of these suggests that they are closer to being 'recipes' than to models which can be adapted to suit individual (classroom) needs. The degree of collaboration involved in the above methods varies, and not all of them incorporate social skills training and group processing. There is a wide variation in terms of reward interdependence and task interdependence and also in terms of individual accountability.

I adopt the Galton & Williamson's, and Pollard's definition of 'collaborative group', and consequently Johnson & Johnson's 'learning together' method. This is the definition I intend to use, for the purpose of this research and which I will simply refer to as group work for now onwards. Thus collaborative group work should not be confused with group work where the students sit together in groups and work on tasks individually, or let one member do all the work. Dunne and Bennett (1990) argue that their effort is to attempt to 'change typical classrooms from children working in groups, to children working as groups' (p. 4).

1.2.2 What is the rationale for collaborative group work?

Any rationale for group work must depend on a coherent theory of learning. The 'constructivist approach' perspective (or 'understanding as active construction of knowledge' according to Putnam, Lampert and Peterson (1990)) is the one which generally holds among mathematics educationists.

Constructivist theory of learning derived originally from the work of Piaget who viewed the child as a source of action and knowledge. Through the complementary processes of assimilation and accommodation and the general developmental process of equilibration the child acquires knowledge. Individuals actively select, retain and transform information to a psychological frame of reference, that is, an internal model or system of representation within which the child makes sense of the world (Gipps, 1992). Whereas Piaget believed that thought and language are closely related but different systems, Vygotsky maintained that thought is internalised language. Vygotsky (1978), whose main writings date back in 1929, believed that knowledge is socially legitimated and children need to be inducted into it. He acknowledged that a discrepancy might exist between solitary and social problem solving when he developed his 'zone of proximal development'. He defined this zone as the gap that exists for children between what a child can do alone, and what he can do with the help of someone more knowledgeable and skilled than himself.

Later on, Bruner (1986) followed Vygotsky by recognising 'that most learning in most settings is a communal activity, a sharing of the culture' (p. 127). He viewed the idea of education as a transmission of knowledge from the teacher to the learner as entirely inappropriate. Consequently in his view the idea of education provides an opportunity for teachers and learners to engage in negotiating a set of shared meanings.

Bruner suggested that we learn from being taught and by what we see others doing, and that language is an important tool for organising children's cognitive structures. Crucial to this idea is that interaction with another person is required, either teacher or peer, to help this process.

Bruner used the term 'scaffolding' (Wood, Bruner and Ross, 1976) to describe the type of support for learning through social interaction. This allows someone to build up his/her understanding alongside another who has greater knowledge.

Edwards and Mercer (1987) write about 'common knowledge' as the understanding created and shared by people through their interaction. They claim that the basis of understanding and learning is 'inherently social, cultural and communicative' (p. 168). Consequently, in their view, learning is essentially collaborative and it develops through people doing things and discussing about them.

Furthermore, they suggest that the interaction and negotiation constitute much more than a mere passing on of information 'when two people communicate, there is a real possibility by pooling their experiences they achieve a new level of understanding beyond that which either had before.'

Mathematics educators such as Cobb, and Confrey started to take into account these views and began to look closely at the implications for mathematics teaching.

Yackel, Cobb and Wood (1991) were guided by a constructivist theory of learning, when they conducted a constructivist teaching experiment involving group work in one second-grade public school to both develop and revise the educational materials and to investigate children's learning. The instructional activities used, had been developed during the first year of the project but they were refined and modified according to the classroom needs from a daily analysis. The starting point for developing these activities was the students' experiences rather than the formal mathematics. Two types of instructional settings, small-group problem solving and whole-class discussion, were developed to facilitate the learning opportunities during communicative interactions in the classroom.

In order for the teacher to gain insight into the children's mathematical thinking, it was essential that the children accept the obligation to explain and justify their mathematical activity and develop the expectation that the activity should be meaningful to them. To bring this about the teacher had to initiate and guide the negotiation of social norms which included that students cooperate to solve problems (listen to each other's solution attempts), that meaningful activity is valued over correct answers, that persisting on a challenging problem is more important than completing a large number of activities and that when working in small groups they explain their solution methods to their partners and try to make sense of their partner's solution attempts.

Mathematics educators have also turned their attention to the process by which peer discussion and collaboration can promote pupils' learning. Hoyles et al. (1991) have explored children's talking in a computer environment, with particular attention to the discussion within the context of collaborative interaction, while Pirie (1991) examining peer discussion in the context of mathematical problem solving argues that

> 'peer group discussion is not necessarily seen a panacea to all errors and misunderstandings, but that if we are to move ahead in this field we

must study with care the discourse between pupils in the natural settings where it occurs and not only that in which the teacher is involved' (p. 161).

Other researchers have been less explicit about their underlying theoretical basis, and promoted group work for a wide variety of reasons.

Good and Biddle (1988) argue that small group instruction is not a panacea but an attractive instructional format which when properly implemented could enable teachers to achieve certain goals: practising meaningfully mathematical topics of appropriate difficulty and interest, learning social skills, adopting different approaches to problem solving, verbalising thoughts about mathematics and enhancing social intelligence.

Schoenfeld (1985) follows Vygotsky in viewing group discussions as 'the social mediating factors that help move the student through a 'zone of proximal development' ... to the point where the appropriate behaviours are internalized' (p. 375). He cites four reasons that justify their use: (1) provide the teacher the opportunity for direct intervention, (2) provoke discussions of plausible solutions, (3) do not harm students and (4) students are individually insecure about their abilities in problem solving.

The NCTM's Curriculum and Evaluation Standards for School Mathematics (1989) states that 'small groups provide a forum in which students ask questions, discuss ideas, make mistakes, learn to listen to others' ideas, offer constructive criticism, and summarize their discoveries in writing' (p. 79).

Barnes and Todd (1977) give an account of their work in setting up small groups, in a variety of curriculum areas, in which students worked together without the supervision of a teacher. Their focus has been concerned with the kinds of talk generated by collaborative, as contrasted with traditional learning situations. Their argument is that small group talk has the exploratory character necessary for personally meaningful learning. Only by thinking aloud, acknowledging uncertainty, formulating tentative ideas, comparing interpretations and negotiating differences, only by these means can learners shape meanings for themselves and others, and thereby arrive at meaningful understanding.

Collaborative classroom work is rooted in a view of learning which rejects the passive role assigned to students by the 'standard' or 'transmission' model. While the 'standard' or 'transmission' view denies the significance of social relationships in the classroom, the collaborative view makes these the very basis of learning. 'To cooperate, to work together, to give up some of one's individual behavior in favor of collective behavior, to collaborate in the pursuit of knowledge and in the search for common good, are the essential goals of the school' (D'Ambrosio, 1990, p. 22).

1.2.3 Is collaborative group work effective?

In the following sections of the literature review the term 'cooperative' instead of 'collaborative', sometimes occurs since it is used in the American literature with the same meaning.

The literature includes increasing discussion on small group instruction in elementary schools. Empirical research illustrates that small group work methods under certain conditions facilitate achievement and affective gains (Slavin, 1989b).

Dunne and Bennett (1990) argue that both children's learning and social relationships can substantially be enhanced by working collaboratively. Johnson and Johnson (1992) explain that more than 520 experimental studies and 100 correlation studies have been conducted the past 90 years comparing the cooperative, competitive and individualistic efforts. Reviewing all these, Johnson and Johnson argue that the following conclusions may be drawn:

(1) 'cooperative efforts resulted in higher achievement and greater productivity',

(2) 'cooperative efforts resulted in greater interpersonal attraction and more social support' and

(3) 'cooperative efforts resulted in higher self-esteem and greater psychological health' (p. 176).

Galton and Williamson (1992) have made an extensive review of the literature on group work from the United States and the United Kingdom. From this they have reached six general conclusions (p. 42-43). What has been learned so far is:

(1) group work does improve children's self-esteem and motivation when they are encouraged to work towards a shared outcome or make individual contributions to a common goal;

(2) groups function best when they are of mixed ability and include pupils from the highest ability group within the class;

(3) 'children perform in different ways according to the nature of the task';

(4) 'problem-solving tasks with a clear testable outcome tend to generate a greater degree of collaboration than more 'open-ended' tasks';

(5) 'for successful collaboration to take place pupils need to be taught how to collaborate so that they have a clear idea of what is expected of them';

(6) 'there remains considerable doubt about the value of building in individual rewards within collaborative exercise'.

There are several reviews of the literature which provide evidence that students' achievement is facilitated by cooperative learning methods (Johnson and Johnson, 1974, Slavin et al., 1985, Slavin, 1989b). In his 'best evidence' review, Slavin (1987) found that 72% of the 68 'adequate' studies showed better achievement effects for cooperative learning than for control conditions (i.e. 49 of the 68 comparisons were positive (72%) whereas only 8 (15%) favoured control groups). He explains that this result appears to be due to combined cooperative and competitive incentives that motivate students to encourage mutual learning. However, reviewers differ sharply about conditions necessary to obtain achievement effects.

1.2.4 What factors / elements are necessary to make collaborative group work effective?

Slavin (1989a) does conclude that positive achievement effects on students learning depend upon the existence of two necessary factors: (1) use of group goals and (2) individual accountability. If these two factors are not present, students have no real interest and share a counterproductive interest in one another's success. Thus, they do not provide one another with elaborate explanations that are essential for the achievement effects. When there are group goals but no individual accountability, students may view interacting with others as a waste of time and may be reluctant to stop to explain concepts to other group members who are having problems. Slavin argues that in this circumstance, the most able students may simply do the work for the rest of the group.

According to Johnson and Johnson (1989) five basic elements must be included for small group learning to be truly cooperative: (1) positive interdependence, (2) individual accountability, (3) face-to-face promotive interaction, (4) interpersonal and small group skills and (5) group processing. Below, these five basic elements of cooperative learning are briefly described.

(1) Positive interdependence: The first element of cooperative learning is positive interdependence. Positive interdependence is achieved when the teacher establishes a system of mutual learning goals (as in Slavin's first criteria), joint rewards, shared materials and information, and assigned social roles within the group. In cooperative learning when groups are structured around positive interdependence, the students help one another, seek help from group members, and allow ideas to evolve through interaction. Students need each other to succeed in completing the group's task. If positive interdependence is to be developed, students within groups must be genuinely dependent upon one another. Artzt (1996) argues that 'the most successful grouping strategies are those that are carefully structured so that a group goal fosters positive interdependence among the members and so that each person is held individually accountable for the work that is done in the group' (p. 117).

There is research evidence that 'cooperative learning experiences promote greater interpersonal attraction and more positive relationships among students than do competitive and individualistic learning experiences' (Johnson and Johnson, 1985, p. 112). In addition, research has shown that positive interdependence is more necessary for achievement to increase, than an individualistic situation where the subjects have the opportunity to interact with others (Johnson and Johnson, 1992).

Bennett and Dunne (1990) showed how task-related talk was much higher in situations where the task was structured so that the children had to work together. In more usual situations, where children sat in groups but worked more individually, task-related talk was less. Overall in their research, 88% of the talk in co-operative groups was on-task compared to 66% in the individual settings.

(2) Individual accountability: The second element of cooperative learning is individual accountability. Every group member should be responsible for his or

her own learning within the group. A successful group outcome should depend on each member taking this responsibility seriously.

(3) Face-to-face promotive interaction: The third element of cooperative learning is face-to-face interaction. Members of the group need to sit facing each other so that they can construct their learning activities through talk involving all group members. Students are expected to discuss the tasks and provide one other assistance and support. In contrast to cooperative work, the traditional system of instruction emphasises the individual work.

Research has shown that the interaction and verbal interchange among students that are promoted by positive interdependence may have a great effect on educational outcomes. Johnson (1981) suggests that constructive student-student relationships contribute to the achievement of educational goals in a number of ways, while Webb (1982) asserts that students interaction is a major factor in determining achievement in cooperative small groups. Further, from this practice students can gain proficiency in social interaction skills which is another element of cooperative learning.

Other researchers have also promoted the importance of students' interaction in learning mathematics. Yackel et al. (1990) describe how they introduced their teaching experiment in second grade classrooms. Small group problem solving followed by whole class discussions was used as a primary instructional strategy. Data suggests that this approach has had success in that students in experimental classes attained more advanced levels of conceptual understanding than those in control classes. Further, project students valued attempts to work hard, understand and collaborate, more strongly than non-project students (Nicholls et al., 1990).

(4) Interpersonal and small group skills: The fourth element of cooperative learning is the appropriate use of interpersonal skills in the group. If students are not used to cooperative work then they will need help with the

communication skills for collaborating effectively with other students. However, I shall treat 'collaborative skills' in a separate section (1.2.5 Learning to cooperate) in which I shall discuss it more extensively.

(5) Group processing: The last basic element of cooperative learning is group processing. This element is crucial to learning the social skills required for cooperative group work. After a cooperative activity students should be given time to discuss how efficiently their groups are functioning and how well they are achieving their academic goals. Noddings (1985) explaining the stages of a research programme on group work states

> 'in our project we have given special attention to the stage of undergoing consequences. Agreeing with both Dewey and Vygotsky on this, we believe it is important for youngsters to 'live through' what they have planned and executed in order that the final stage of evaluation may be truly reflective. We, therefore, provide a debriefing period at the end of each problem solving session' (p. 354).

Nevertheless the research into cooperative learning that correlates observed interaction with achievement is to a large extent inconsistent. For example, while many researchers claim that interaction in small groups (under certain conditions) facilitates achievement, others find no evidence to support this view. Webb (1991) reviewed the research linking task-related verbal interaction to learning in small groups in mathematics classrooms and concluded that giving elaborate (content-related) explanations is positively related to achievement (in 11 of the 15 correlations) whereas providing other kind of help is not related. Moreover she found that receiving content-related explanations did not seem to be beneficial for achievement. In only 3 of the 14 partial correlations in the studies surveyed, did receiving elaborate explanations produce positive effects on achievement. However, receiving help other than content-related explanations was either related negatively to achievement (non-responsive feedback) or not conducive to achievement (responsive feedback). These findings suggest that examining group work should involve the extent to which students are encouraged to provide explanations to others.

The most positive predictor of achievement in Webb's review is the providing of detailed, elaborate explanations. In other words the student who does the explaining is the one who benefits. Swing and Peterson (1982), qualifying Webb's position, found that high-achievers benefited from participation in heterogeneous groups, especially by giving explanations to others. Moreover, students with higher initial achievement scores tend to give more explanations. Providing more detailed explanations is, in turn, related to the student's conception that better explanations are those that include specific content or information (Peterson and Swing, 1985). These concepts of a good explanation are significantly related to group achievement on seatwork with arithmetic tasks (Peterson and Swing, 1985).

Webb's (1991) review also suggests that group composition is important in the kind of interaction that will occur. She examined and analysed the results of few studies which have compared several of five group compositions: (1) mixed-ability groups with high-ability, medium-ability, and low-ability students, (2) mixed-ability groups with highs and mediums or mediums and lows, (3) homogeneous groups with high-ability students, (4) homogeneous groups with medium-ability students and (5) homogeneous groups with low-ability students. Webb concluded that mixed-ability groups with a wide range of ability are beneficial to relatively high-achieving and low-achieving students but not to medium-achieving students. Moreover she concluded that interaction in mixed-ability groups with high-ability and medium-ability students or medium-ability and low-ability students as well as in homogeneous groups with medium-ability students appeared to be beneficial to most group members.

1.2.5 Learning to cooperate

Many studies of group work have observed that groups quite frequently fail to show behaviours that could be called cooperative. Therefore these studies suggest that training of pupils in cooperative skills is necessary. Training in the skills for working successfully with others might entail knowledge of how to listen, question, challenge, help and give explanations to others (Bennett and Dunne, 1990). Both Hertz-Lazarowitz (1992) and Johnson and Johnson (1992) regard the development of interpersonal skills as one of the necessary conditions for effective working in cooperative groups. Galton and Williamson (1992) argue that 'children need to be taught how to collaborate by breaking down activities into small-scale exercises to improve certain competencies and skills, such as listening and handling disagreements' (p. 120).

Research particularly in the United States has shown that when such skills are practised the quality and effectiveness of group work improves. Webb (1989) found that the most effective groups display high levels of elaboration and explanation of problems in their discussions and group members can be trained to provide this type of interaction.

Swing and Peterson (1982) experimented with training fifth-grade pupils in task-related interaction and more specifically, in improving explaining skills. They therefore developed a two-session training programme. The first session used guidelines for interaction and the second session included a practice in explaining. In the second session each student had the chance to explain a problem and to receive feedback from training personnel. The results indicated that, compared with a control group of students, those who received the training programme gave more explanations and checked each other's answers more frequently. Although there were no differences in achievement between the two conditions, trained students with low scores on the pre-test outperformed control students with similar scores on the pre-test. The results of the Swing and Peterson (1982) study do suggest that direct training exercises can have positive effects on small group interaction.

1.2.6 Criteria for forming groups

In Pollard's (1997) view criteria by which groups may be formed may include the following:

Age: These groups can be formed for some activities but not for specific teaching points because of the differences in ability, interest and needs of the pupils.

Attainment: Forming groups on the basis of students' attainment are useful for specific tasks. Webb (1991), however, explains that homogeneous ability groups may have positive and negative results. According to her, a study showed that homogeneous high-ability students made little effort to provide explanations to their group-mates, while homogeneous low-ability students tended to offer incorrect explanations because of the lack of sufficient skills. In Webb's view, further studies showed that the only homogeneous ability groups which were very active were those of medium-ability groups. Students in homogeneous medium-ability groups gave more explanations, received more, and demonstrated higher achievement.

Interest groups: This form of grouping is based on students' common interests, and it can be employed from time to time. Pollard stresses that it can have positive results when the class is composed of children of different attainment, race and social class.

Friendship groups: These groups provide opportunities for social development. On the other hand they may lead to ability and gender biased friendships. However the teacher should be aware of any isolated children in class, as friendship groups do not encourage the integration of marginal children.

1.2.7 Peer tutoring

According to Damon and Phelps (1989) peer tutoring 'is an approach in which one child instructs another child in material on which the first child is an expert and the second a novice' (p. 11). Although peer tutoring requires the roles of the tutor and tutee to be clearly defined and carefully specified, it may occur informally in cooperative learning through positive interdependence, and thus a brief reference is made to supporting literature. Peer tutoring could also be understood through the social interaction view of cognitive development.

Topping (1998) examining research on peer tutoring (in schools) explains that there is substantial evidence that this is effective in schools. Reporting the findings of a meta-analysis of studies, conducted by Cohen et al. (1982), he explains that in 65 studies with control groups, tutored students outperformed controls in 45. Moreover, peer tutoring improved tutee attitudes in class, as well as their self-concept. Topping also reported another meta-analysis of 82 studies in schools, conducted by Sharply (1981), in which substantial cognitive gains for both tutees and tutors were presented.

1.2.8 Group roles

Ann Gold (1998) discussing Beldin's (1993) work, explains that Beldin looking at the most useful people to have in teams (with adults), found that there were some important roles that are essential to the functionality, creativity and productivity of a team. The team roles he identified are: (1) plant, (2) resource investigator, (3) chair, (4) shaper, (5) monitor evaluator, (6) team worker, (7) implementer, (8) completer, (9) specialist.

According to Gold, Beldin lists these nine team roles, describing also personality traits for each one, while he stresses that combination of these characteristics may exist in one person. In other words it is not necessary to have nine people in every team, but successful teams should have members who are able to operate and influence in all these aspects. An analogy of the team roles could exist in the classroom, in the form of a group, where students work together to accomplish a collective task. When the members appear not to function as expected this may show that some role is missing. Conversely, when a group functions successfully this may be an indication that some of the above roles may be satisfied.

Bennett and Dunne (1992) elaborating on key roles for group members, explain that in a group of four an individual could be assigned one of the following roles: (i) co-ordinator, (ii) data gatherer, (iii) secretary, (iv) evaluator.

1.2.9 Attitudes and behaviour

Various aspects of attitudes and behaviour

McLeod (1992) in an article of the research on the affective domain in mathematics education, suggests that there are at least three major facets of the affective experiences/responses: (i) beliefs, (ii) attitudes and (iii) emotions.

(i) Beliefs are cognitive in nature and generally stable.
(ii) Attitudes are affective responses that 'involve positive or negative feelings of moderate intensity and reasonable stability' (p. 581). Examples of attitudes toward mathematics would include liking exercises of basic operations or algebra and disliking word (story) problems or geometric proofs.
(iii) Emotions are non-cognitive and short-lived as they may appear and disappear quickly. Students emotional reactions have not been a well researched area.

Self-confidence - self-concept - self-efficacy

In addition to the responses of beliefs, attitudes and emotions, there are other research areas of the affective domain that have implications for mathematics teaching and learning. Self-confidence, self-concept and self-efficacy are three of these areas which appeared to have particular interest in my research.

Self-confidence in mathematics is related to how competent a person believes themself to be in mathematics. Research has produced some interesting results about self-confidence in mathematics and mathematics achievement.

Self-confidence is related to another affective experience, self-concept. According to Reyes (1984) self-concept can be thought of as a generalisation of confidence in learning mathematics. It is the student's perception of his own learning competence.

Kutnick (1988) argues that building up a pupil's academic self-concept is a complex process, more generally a combination derived from teachers interventions and the pupils' performance.

Pollard (1985) attributes pupils' behaviours as a manifestation of the pupils' attempt to maintain their self-esteem thereby enhancing their self-concept. In this meaning self-concept is a global judgement based upon individual self-images or beliefs about themselves, and self-esteem is the evaluation of this belief.

Closely related to self-concept is the notion of self-efficacy. Bandura (1982) argues that a person's belief about their abilities to engage in activities necessary to allow designated performance levels is critical to a pupil's self-efficacy and is highly likely to influence their choice of activities. In line with Bandura, Schunk (1984) points out that self-efficacy is related to decisions about which activities students will choose to participate in, how much effort will exert and how long they will persist in those activities.

McLeod (1992) examining the research on self-efficacy points out that 'although the data on self-efficacy are interesting, it is difficult to sort out why self-efficacy as a construct should be more successful as a predictor than mathematical self-concept or confidence in learning mathematics' (p. 584).

Most of the intervention studies that have been reported in the mathematics literature indicate that students have improved their attitudes towards mathematics and they have increased their mathematics achievement (Osborne et al., 1997). In particular, studies which have been concerned with the improvement of the experiences of all students, have changed the way of teaching mathematics. They tended to make mathematics teaching more experiential, use more problem solving activities and employ constructivist theories of learning (Cobb et al., 1991). Two of these intervention studies involve group work.

In a year long study by Cobb et al. (1991), quoted previously in section 1.2.2 (as Yackel, Cobb, Wood (1991)) ten second grade classes were taught mathematics using problem solving activities (in group work), and eight second grade classes, control classes, were taught mathematics using textbooks. At the end of the school year the two groups of students (project and non-project) were compared on a standardised achievement test and on instruments designed to assess their conceptual development in arithmetic, their personal goals in mathematics and their beliefs about reasons for success in mathematics.

The results indicated that project students differed in their views about mathematics and some of these differences were statistically significant. The project students saw less value in being superior to their peers in mathematics, they saw less value in conforming to teacher's or peers' solution methods, they saw less value in being lucky, neat or quiet in class, and they valued attempts to understand and collaborate more strongly than non-project students.

At the end of the school year the students in the project groups were assigned to textbook-based classes. A year later they were tested again and the researchers found the following significant differences: Former project students attained more advanced levels of conceptual understanding, they held stronger beliefs about the importance of working hard and being interested in mathematics and they held stronger beliefs about understanding and collaborating. In addition they attributed less importance to conforming to the solution methods of others.

Thompson and Thompson (1989) designed a study to explore affect and problem solving in an elementary school mathematics classroom. The purpose of the study was to examine (i) the practice of a teacher focused on problem-solving with students working in groups, and (ii) the affective states of three students in the teacher's class. The three students were selected on the basis of their scores on an instrument to measure students' perceptions of their own mathematical aptitude and attitude. Each student was observed and videotaped during one problem solving session. All (three) students were interviewed following the class session. The data collected from this study, suggests that there was a relaxed climate in class during problem solving and this can be attributed to two aspects of the instruction: (i) the use of the small groups and (ii) the teacher's behaviour. The use of small groups served to reduce the anxiety that students would have faced if they were to solve their problems individually. On the other hand the teacher's friendly behaviour and his tendency to accept students' suggestions unquestioningly appeared to help students open up, creating a low stress atmosphere during problem solving. However, the researchers expressed some concerns about the teacher's behaviour, regarding the need for students' explanations.

1.2.10 Implications for this study

My aim was to introduce group work in order to improve attitude and improve learning. The reason I chose the collaborative form of group work to achieve this aim is based in a theoretical position (1.2.2 What is the rationale for collaborative group work?) and also in an empirical position (1.2.3 Is collaborative group work effective?). These sources suggest that collaborative group work should be used, rejecting all the other types of group work cited by Galton & Williamson (1992) and Pollard (1997) (see section 1.2.1).

But the question arises as to what particular form of collaborative group work should I use? After looking at Slavin's listing I chose the Johnson & Johnson's 'Learning together' method and I rejected all the others.

The first two collaborative methods (i.e. 'Student team learning' and 'Jigsaw') seemed too rigid, as they were less focused on sharing knowledge because of the task demanded and the contribution to the intended outcome. The students work in groups but they do not really share knowledge contributing to a single joint outcome, as they may work on different units (TAI, one of Slavin's methods), or they may be assigned a unique section of a material that has been broken into sections ('Jigsaw' method). The third collaborative method 'Group investigation' was too open to use in the Greek syllabus. Therefore, the 'Learning together' method was selected because, as I already explained, it seems to be better supported by the empirical research and also it seems to better meet the theoretical background.

The 'Learning together' method is a different approach from the other collaborative methods. Its arrangement is much closer to the collaborative group (4^{th} type) as defined by Galton and Williamson above (see figure 1.1). Unlike Slavin's own approach ('Student team learning' method), where individuals complete the assignment by themselves and the scores of individuals are then aggregated, Johnson & Johnson recommend that the outcome of the group's activity should consist of a single assignment sheet.

An additional advantage of the 'Learning together' method is that there are no clearly defined procedures. It is rather a method which the teacher can adopt and adapt in a variety of teaching situations. The emphasis is very much on the teacher to plan his/her own strategies, within the framework of setting up objectives, defining the task, structuring the goal, reward and task interdependence, determining the criteria for success and specifying desired behaviours.

The collaborative 'Learning together' method which I selected requires that learning groups of mixed sex and ability be established and no specific leader be appointed. Students work on the same problems and they have a shared goal. Collaborating with their group-mates, students are expected to develop meaningful relationships and learn to manage conflicts constructively. The group function is emphasised and the social skills are directly taught. Making my expectations explicit to the students seemed essential (Johnson and Johnson, 1989, 1994).

Although not every collaborative learning group will necessarily meet all these criteria, I think each group should be heterogeneous in relation to gender and have a shared goal.

However, in deciding about the range of attainment for the formation of groups, I accepted Webb's work because of the evidence provided by the empirical work (Webb, 1989, 1991) and also because it seems to be a valid explanation (i.e. based on the finding that the student who is doing the explanation is the one who benefits, and also that in wide-range attainment groups the medium-attainers tend to be left out of the group interaction because the high-attainers and the low-attainers form a teacher-learner relationship). Other researchers have used wide-range attainment groups, but I adopted Webb's position because she has researched most this area. Although my aim was to use narrow-attainment range groups (Webb's position), for practical reasons this was not always possible (see section 3.3.1).

Furthermore, part of my main aim was to explore possible changes in the students' attitudes as they move from a traditional class teaching to a group working environment (Cobb et al., 1991, Yackel et al., 1991).

Therefore, the implications of the above literature review for using group work in my research were the following:

- choosing (and establishing) collaborative group work
- explaining to students the relevant social skills and training them in these skills
- attempting to narrow the range of attainment in groups
- exploring changes in pupils' attitudes.

1.3 Word problem solving

In this section I review the relevant literature on word problem solving. An examination of the role of the word problems in the students' math textbooks' and in the teachers' guides in the Greek primary school is provided.

1.3.1 Problems and problem solving

Several alternative definitions of the term problem have been proposed at different times. A typical one is:

'A person is confronted with a problem when he wants something and does not know immediately what series of actions he can perform to get it' (Newell and Simon, 1972, p. 72).

Similarly problem solving is typically defined as:

'It is the process by which the individual uses previously acquired knowledge to resolve the problem which confronts him or her' (Krulic and Rudnick, 1988, p. 123).

'Finding a 'solution path' (rather than simply following one) is the essence of problem solving' (Burkhardt and Schoenfeld, 1988, p. 3).

For the purposes of this research I shall use the following definition, which seems to be descriptive of, and suited to mathematics problems. It has been provided by Henderson and Pingry (1953) and includes all the conditions mentioned in the above definitions.

Henderson and Pingry have provided three criteria for what they call 'a problem for a particular individual'. The criteria are:

'1. The individual has a clearly defined goal of which he is consciously aware and whose attainment he desires.

2. Blocking of the path toward the goal occurs, and the individual's fixed patterns of behavior or habitual responses are not sufficient for removing the block.

3. Deliberation takes place. The individual becomes aware of the problem, defines it more or less clearly, identifies various possible hypotheses (solutions) and tests them for feasibility' (p. 230).

From the above definitions of the terms problem and problem solving a teaching corollary is obvious: It is not always easy for the teacher to know when a task will be a problem for the students. The situation is complicated by the fact that what constitutes a problem for one person may not be a problem for another. 'A problem for one child may be just an exercise for another' (Alexander et al., 1988, p. 51), and this is maybe because of children's differences in knowledge, skill, experience, ability, and interest. For example, two students may be both interested in solving the same situation with the following distinction. The first student has a routine process that has been previously learned and it can be immediately applied to solve the problem, while the second student lacks the knowledge of such a routine procedure. In this case, the situation is not a problem for the first student as it does not contain anything new but would pose a problem for the second student.

1.3.2 What is a word problem ?

A wide variety of mathematical situations are included in the elementary school mathematics curriculum. Among these types of mathematical problems is the word problem or story problem: a verbal or written description of a 'real' situation involving numerical relationships and requiring mathematical analysis. The situation and relationships must first be interpreted, understood and then analysed. Then arithmetic algorithms may need to be carried out to arrive at the answer.

Word problem solving is 'an important component of the elementary school mathematics curriculum' (Rathmell and Huinker, 1989, p. 99). The expectation is that students must learn to solve word problems in schools to cope with problematic situations of their school, family and social life. Scieszka and Smith (1995) indirectly imply that we can think of everything in our life as a maths problem. But in everyday life problems are not posed in the form of exercises and students will have to first make sense of the problematic situation, try to relate to it and then analyse it. However, word problem solving may help students to engage in problematic situations and realise how mathematics can be applied to resolve them. Rathmell and Huinker (1989) argue that

> 'word problems offer a context in which a rich variety of meanings for the basic arithmetic operations can be developed. Children should have a broad conceptual framework of understandings for each of the operations so that they can apply them meaningfully in a variety of situations. Word problems serve as a setting for the experiences that lead to these broader understandings' (p. 1).

Although there appears to be no disagreement among (maths) educators as far as the definition of word or story problem is concerned, there are different views for the purpose of the word problem. One view supports that '(word) problems are an important part of the curriculum inasmuch as they represent the interplay between mathematics and reality and give a basic experience in mathematizing' (Sowder, 1989, p. 148). In other words the purpose of word problem solving for the student appears to be the translation of a real situation into mathematical terms as well as experiencing the relationship of mathematics to reality. This view implies that word problems may be viewed as a cultural device for representing reality in a symbolic form.

However, Lave (1992) takes a different view and distinguishes problems of sense and problems of scale. The problems of sense can be addressed by persons in activity, while the problems of scale occur in situations in which persons have no access to activity. Lave argues that word problems often turn problems of sense into those of scale thus destroying the claim that they reflect everyday experience.

Moreover, other researchers have carried out review of studies in order to examine the role of word problems in the primary school (see Verschaffel and De Corte, 1997).

There are strong arguments for teaching word problem solving. It would be wrong, however, not to consider the negative side of such an approach, for there are certain practical difficulties that cannot be treated superficially. The most serious of these is the insecurity that teachers feel about teaching word problem solving. 'Many teachers do not feel very successful in teaching story problems' (Davis and McKillip, 1980, p. 81).

At this point I wish also to refer to the role of key words in attaining solutions to word problems, because this strategy was used by both teachers and students involved in this study. Laborde (1990) argues that among the variables that may influence the students' representation of problems and therefore their solution strategies, is the verbal formulation of the problem. In many cases, certain key words (e.g. 'altogether', 'left', 'more', 'less') of the problem's text may lead the student to the required mathematical expression but in other cases these same words may divert him/her from the solution. Thus, the wording of the problem may have a positive or negative result on students' solution method.

Following Jerman's (1972) distinction between 'verbal cues' and 'distractors' Nesher and Teubal (1975) explain that

> 'there are really numerous instances in which the key words in the verbal formulation can serve as 'cues' which lead directly to the mathematical operation required for the solution. However, there are many other instances in which the same key words might act as a 'distractor': i.e., might lead the solver to a mathematical operation (currently associated with them) which is not required for solution' (p. 43).

1.3.3 Classifications of word problems

Several classifications of word problems have been proposed at different times. Below I cite those that have been suggested by two different academic teams. The first classification consists of two types of word problems while the second consists of six types. Most classifications tend to be in line with the second of these.

LeBlanc, Proudfit and Putt (1980) distinguish two types of mathematical (word) problems in the elementary school curriculum, (1) the standard textbook problem and (2) the process problem:

(1) The standard textbook problems may be solved by the direct application of one or more operations or algorithms that have been previously discussed and

learned. 'The purposes of standard textbook problems include improving the recall of basic facts, strengthening skills with the fundamental operations, algorithms and reinforcing the relationship between the operations and their applications in real-world problems' (p. 105).
(Example: *Chocolates come in packs of 4. A carton holds 36 packs. How many chocolates are in the carton?*)

(2) Process problems require the use of one or more strategies or even some non-algorithmic approach. 'They emphasize the process of obtaining the solution rather than the solution itself' (p. 105). Process problems aim to the development of problem solving strategies providing students the opportunity for critical thinking and creativity in their solution methods.
(Example: *Twelve teams participated in a soccer tournament. If every team played one game against each other team, how many games were played?*)

Charles and Lester (1984) discussed different types of mathematical problems in the school curriculum and they considered six situations: (1) drill exercise, (2) simple translation problem, (3) complex translation problem, (4) process problem, (5) applied problem, and (6) puzzle problem.

In the following paragraphs I shall give a brief description of the middle four situations, and I shall leave out the first and the last situation as they are irrelevant to word problems as defined. This is because drill exercise is not a word problem, and puzzle problem although it takes the form of a word problem, its solution depends 'upon luck, insight or unusual strategies' (Askew and Wiliam, 1995, p. 22).

(i) Simple translation problem: This is a very common type of word problem in maths textbooks. The solution requires translating the words into a mathematical expression. The primary purpose of a simple translation problem is to increase and reinforce students' understanding of mathematical concepts, to help them maintain proficiency in computational skills and to illustrate real-world situations that involve mathematical skills and concepts.

(Example: *Peter went to the supermarket to buy 4 crates of orange juice. If each crate contains 6 bottles, how many bottles of orange juice did Peter buy?*)

(ii) Complex translation problem: This type of word problem is similar to the simple translation problem except that its solution involves at least two operations and thus is called complex. The purpose of this type of problem is similar to the previous one (simple translation type).

(Example: *Pencils come in packs of 10. A box holds 20 packs. The owner of a bookstore ordered 2.600 pencils. How many boxes did the owner order?*)

(iii) Process problem: This is a type of problem for which there is no obvious algorithm that will give the correct answer. Its solution requires the use of one or more thinking strategies. Since the emphasis here is on the process of solving the problem, it is called process problem. The purpose of this type of problem is 'to develop general strategies for understanding, planning and solving problems, as well as evaluating attempts at solutions' (Charles and Lester, 1984, p. 10).

(Example: *There were 14 guests at a party. If each person shook hands with everyone else, how many handshakes were there in total?*)

(iv) Applied problem: This is a genuine real-world problem or at least a realistic situation. Its resolution requires the use of mathematical skills, concepts, facts, and procedures. This type of problem provides an opportunity for students to appreciate the usefulness of mathematics in everyday life. Characteristic of the applied problems is that there is generally no single clear 'right answer' or there is no single answering procedure and this renders many teachers of mathematics uncomfortable with real problem solving.

(Example: *Is it better to use a white-board or a blackboard?*)

Comparing the types of word problems that have been proposed by Leblanc, Proudfit and Putt (first group) and by Charles and Lester (second group) I think there is no significant difference between them. What Charles and Lester call simple and complex translation problems belongs to the standard textbook problem for the first group (who defines it as the problem that 'can be solved by the direct application of one or more previously learned algorithms', p. 105). A process problem is defined in a similar way from both groups. The only divergence is that the second group presents another type of word problem called applied. This type is very rarely met in maths textbooks and especially not in primary education.

Burkhardt (1981) examining mathematical modelling offers a slightly different classification of word problems. This is based on students' interest in the problem, which is seen by Burkhardt to be a major factor. His categorisation overlaps with Charles and Lester but introduces the term 'dubious problem' as an alternative description for whatever Charles and Lester have called simple and complex problem.

Dubious problems are the problems whose main purpose is to provide us practice in mathematical procedures.
(Example: *Nick calculated and found that he is aged 108 months. Then he thought that together with his sister, both complete 23 years. How old is Nick's sister?*)

The most common type of word problem found in maths textbooks for primary education is the standard or routine problem. But it is not a problem in the true sense as it does not contain anything new for students. Standard problems often appear at the end of units, and play the role of simple exercises, where students know the operation to use usually from the unit's title.

Although this is a pattern established in most primary maths textbooks (and teaching approaches) 'there is substantial research evidence that demonstrates that treating standard or story problems as problems - not assuming that pupils will instantly spot which operation to use - can provide effective learning situations' (Askew and Wiliam, 1995, p. 23).

1.3.4 Greek research in word problem solving

Empirical research in Greece (Kothali and Georgakakos, 1992, Basetas, 1995) has been conducted with students both from primary and gymnasium and it has confirmed students' deficiencies in word problem solving.

Kothali and Georgakakos (1992) conducted empirical research to examine how students treat word problems in primary school as well as at their enrolment into gymnasium. Their sample consisted of 269 primary students (grade-four and grade-five) and 383 gymnasium students (first-grade). The researchers designed a test comprising five word problems. The first three were routine (standard) problems of one step , i.e. they required for their solution only one operation. The fourth problem was also a routine problem but of many steps, with additions and multiplications, while the last one was a non-routine problem. The test which was administered to primary students included only the first four word problems (standard), and the test which was given to gymnasium students had all (five) word problems.

The results of the research show that in general the percentage of primary students who fail to solve the fourth problem, that combines two operations, even simple ones (addition - multiplication), is quite high (39% for grade-four and 32% for grade-five). Also the results for the gymnasium did not appear very optimistic. In the fifth problem, the non-routine problem, which was administered only to the first grade of the gymnasium, 111 students from 383 answered all the questions in the problem correctly, i.e. a percentage of 29%, while 80 from 383 students solved correctly all of the five problems, i.e. a percentage of 21%.

In another empirical research project, Basetas (1995) examined the ability of leavers from primary school in solving operations and word problems with integer numbers. It was found that in a sample of 610 students from grade-six of the primary school and grade-one of the Gymnasium, a percentage of 12% solved correctly all the computations in the four basic operations and a percentage of 15% solved all the word problems.

Partly, in order to respond to these results the curriculum was revised (1993) for the grade 4, 5 and 6 classes. In line with this revision, parts of the more difficult mathematical content were replaced by other exercises that emphasised calculation skills and word problems (Adamopoulos, 1993a, 1993b).

CHAPTER 2 THE DESIGN OF THE RESEARCH

This section concentrates on the methodological approach, which was employed in conducting my research. The working scheme used a case study approach (see section 2.2) with some characteristics of a teaching experiment and some of action research (see section 2.12).

2.1 Key questions

As I have already outlined, this research project aimed to explore changes in the students' attitudes and behaviour while moving from traditional class teaching to group work. This was explored in the context of word problem solving in two fourth-grade classes in a Greek primary school in England.

There are a number of clear questions emerging from this general aim, which I wanted to try to answer as they arise within this case study.

- Does the students' way of working change in collaborative group work?
- Do students' attitudes to word problems change during collaborative group work?
- Do mixed ability groupings facilitate collaboration, and the teaching and learning of word problem solving?
- Is collaborative group work an effective way of learning word problem solving?
- How do the students' attainments change as a result of this approach?
- Does working in a collaborative environment require a particular role for the teacher?

2.2 The case study approach

From the nature of the questions in section 2.1 it seemed appropriate to investigate in depth possible changes of the students' attitudes and behaviour when they move from traditional class teaching to group work. Conducting case study research rather than a large-scale survey would be a way of producing empirical data of rich and analytical descriptions which would illuminate more effectively students' attitudes and behaviour. So I decided to look in detail at individual students in a small number of classes, rather than to obtain a general overview of a large sample of typical classrooms.

According to Bassey (1999) there are two kinds of empirical educational research: studies of singularities and studies of samples. The first, to which my study belongs, embraces experiments, case studies and action researches, and the second, surveys. So my research is a study of singularities, which uses a case study method with some aspects of action research and some aspects of teaching experiment (i.e. it combines all three methodologies to some degree).

Simons editing the contributions of the conference 'Methods of case study in educational research and evaluation' into a book entitled *Towards a Science of the Singular*, noted that case study

> 'has antecedents in the disciplines of sociology, anthropology, history and psychology and the professions of law and medicine, each of which developed procedures for establishing the validity of case study for their perspective purposes. But the use of case study in education has been comparatively recent; its specific relevance to education has not been explored to the same degree' (Simons, 1980, p. 1).

Adelman et al. (1980) reflected on this conference listing the 'possible advantages of case study' (p. 59-60):

'(a) Case study data, paradoxically, is 'strong in reality' but difficult to organise. In contrast other research data is often 'weak in reality' but susceptible to ready organisation ...

(b) Case studies allow generalisations either about an instance or from an instance to a class. Their peculiar strength lies in their attention to the subtlety and complexity of the case in its own right.

(c) Case studies recognise the complexity and 'embeddedness' of social truths. By carefully attending to social situations, case studies can represent something of the discrepancies or conflicts between the viewpoints held by participants. The best case studies are capable of offering some support to alternative interpretations.

(d) Case studies, considered as products, may form an archive of descriptive material sufficiently rich to admit subsequent reinterpretation ...

(e) Case studies are 'a step to action'. They begin in a world of action and contribute to it. Their insights may be directly interpreted and put to use ...

(f) Case studies present research or evaluation data in a more publicly accessible form than other kinds of research report, although this virtue is to some extent bought at the expense of their length.'

Hammersley (1992) suggests that 'the choice of case study involves buying greater detail and likely accuracy of information about particular cases at the cost of being less able to make effective generalisations to a greater population of cases' (p. 186). Despite the advantages of the case study, it is usually weaker in the generalisability of findings. Supporting Hammersley's view, Elliot

(1992) quotes Simons' (1978) explanation that case studies are low in generalisability.

However, one could equally agree with Alison (1985) who by referring to Sharples (1983) argues that 'an enhanced understanding of the particularity of a teacher's situation is more important than generalizability, and that replicability and transferability are less important than authenticity and accountability' (p. 131).

The above question between the uniqueness and the generalisation of a case study has been the theme of a recent paper by Simons entitled 'The paradox of a case study' (1996). In this paper she addresses the paradox between the study of the singularity and the search for generalisation:

> 'One of the advantages cited for case study research is its uniqueness, its capacity for understanding complexity in particular contexts. A corresponding disadvantage often cited is the difficulty of generalising from a single case. Such an observation assumes a polarity and stems from a particular view of research. Looked at differently, from within a holistic perspective and direct perception, there is no disjunction. What we have is a paradox, which if acknowledged and explored in depth, yields both unique and universal understanding.
>
> Paradox for me is the point of case study. Living with paradox is crucial to understanding. The tension between the study of the unique and the need to generalise is necessary to reveal both the *unique* and the *universal* and the *unity* of that understanding. To live with ambiguity, to challenge certainty, to creatively encounter, is to arrive, eventually, at 'seeing' anew' (Simons, 1996, 225, 237).

In designing my research, I asked myself how it would be possible to obtain an authentic account of classroom life, without forcing the data into a theory and avoiding the temptation of 'hammering the results into shape' (Hitchcock and Hughes 1989). Clearly, I had to find methods that would enable me to seek answers to the initial questions, to produce data that was valid in offering a true reflection of what went on, and was reliable in offering a method of data collection that is replicable. For these reasons, this case study takes mostly an interpretative qualitative rather than a quantitative approach, primarily as a need for understanding students' attitudes and behaviour when they move from traditional class teaching to group work.

2.3 The choice of site for the case studies

As I wanted to work within a Greek context using the Greek syllabus, I decided to conduct my research in a Greek primary school in England because I was based in England. The collection of data for my research took place in two fourth-grade classes (the students' age was 9 to 10 years old) of this school. This is an ordinary school that operates according to the laws and the jurisdiction of the Greek Ministry of Education as do all the public schools in Greece. The majority of students attending this school are of Greek origin and their parents are working in England. Consequently, in a sense, these students are considered to be 'international' in comparison to their Greek counterparts.

This Greek school is a day time school and all the subjects are taught in Greek. English is taught as a second language. Although the context will be different in the two settings (Greece - England), what is more important is that the social and the cultural environment (and also the background) of children will be more or less the same. The students develop relationships with their classmates based on Greek customs and habits. When they work in class they think of Greece and make reference to it. Characteristic was the knowledge of current events in Greece and that some did not speak English at all. They are familiar with the culture contained in mathematical word problems and they follow them easily. In general, comparing a school in Greece with this particular school I would say that there are no differences, except from the countries in which the schools are located.

However, I should point out that many of the students of this school have been under some kind of emotional disturbance because of their move from Greece to England. This is what differentiates them from those in a typical school in Greece.

2.4 The contextual aspects of the research

The contextual aspects of my research were (1) the Greek syllabus, (2) the Greek maths textbook used in grade-four and (3) the word problems included in the relevant textbook.

(1) The Greek syllabus: As the aim of my research was to explore the students' attitudes and behaviour in two fourth-grade classes in a Greek primary school, where the students work in groups to solve problems, I limited my research to the Greek syllabus.

(2) The Greek maths textbook used in grade-four: The centralised educational system in Greece does not provide the teacher the opportunity to choose an alternative book. Therefore, all the students who are in the same grade, all over the country, are taught from the same maths textbook.

(3) The word problems included in grade-four textbook: In the first case study (teaching experiment-1), one limitation concerned the kind of word problems used for learning activities. In all sessions of the teaching experiment-1, I used the textbook's word problems since the regular classroom teacher and the students' parents were very concerned with covering the syllabus for grade-four. My attempts to use slightly different word problems from those commonly encountered in grade-four textbook elicited strong reactions by the parents. One mother told the teacher that her son was very disappointed because he

could not solve the problem I had written on the board (I had added some extra information which in fact did not affect the problem's solution). In another case a father expressed some concerns to the teacher about the grade-four class not covering the prescribed maths syllabus for the year and he threatened the teacher that next time he will report that to the principal. The conclusion from a discussion I had with the teacher was that in order to avoid further problems I was to conduct my research using the word problems in the textbook.

However, this limitation was not applied in my second case study (teaching experiment-2). When I asked the classroom teacher if I could use additional material (word problems), he replied: "Of course you can." and he explained to me that "The students are accustomed to this as I usually assign them additional problems to work on at home."

2.5 Word problems in the Greek primary school

From 1982 a new mathematical curriculum and new maths textbooks were introduced into primary schools in Greece (Troulis, 1992, p. 36). The curriculum provides the guidelines for teaching and learning. For each unit in the textbooks, the authors have defined specific objectives which determine with analytical behaviouristic precision all the teaching actions and the learning activities. In the next paragraph I shall describe the instructions provided by the Pedagogical Institute, in the teacher's maths guides, for a unit from grade-six.

In the second revised volume of grade-six maths textbook (Ministry of National Education and Religion, 1993c, v. 2, p. 14-18) three consecutive units (4-5-6) appear under the titles: 'We solve problems (1)', 'We solve problems (2)', 'We solve problems (3)'. In the teacher's maths guide the Pedagogical Institute has specified for these three units the following objectives, teaching actions and students' activities:

'OBJECTIVES

The units 4-5-6 aim:

- To activate the students' inventive abilities and to help them in problem solving through the following procedure:

  Understanding the problem, finding the connection between the given and the unknown, designing a plan for solution, executing it, and checking the answer.
- The students practise in order:

  a. To state and solve their own problems, similar to those they have solved.

  b. To solve problems which include diagrams.

TEACHING ACTIONS AND STUDENTS' ACTIVITIES

The problem on page 14 of the textbook or another similar, is assigned to the students. For its solution the methodology which is suggested on the same page is followed step-by-step' (Ministry of National Education and Religion, 1994, p. 19).

Characteristic in the strategy of the curriculum is the linear order of mathematical knowledge. The content of the mathematical knowledge in the textbooks of primary school is broken up into components, chapters and lessons. This manner of itemising the teaching knowledge provides for the teacher ready-made lesson plans/activities on a weekly and yearly base. This strict separating of knowledge into chapters and units does not stimulate the students. It also leads to the imposition of technical control (Mavrogiorgos, 1985, p. 30) on teaching practice and to the establishment of uniformity. This type of practice discourages teachers from applying pedagogical innovations.

2.6 Word problems in the students' maths textbooks

In the maths textbooks for grade-four and grade-six there is a suggested model for solving word problems. This model consists of three steps and is basically similar for both grades. Minor differences appear within the activities included in each problem solving step. In the grade-four maths textbook (Ministry of National Education and Religion, 1993a, v. 2, p. 54) the problem solving model appears with the following structure:

'Step 1: We find what is asked for and the operations that we need to perform to solve the problem.
Step 2: We execute the operations.
Step 3: We answer (the problem).'

In the grade-six maths textbook (Ministry of National Education and Religion, 1993b, v. 1, p. 14) the model takes the following form:

'Step 1: We study the problem.
We find the question.
We find the necessary information for solving the problem.
Step 2: We find the operations that we must execute for solving the problem.
We carry out the operations.
Step 3: We answer the question and we check if the answer is within reason.'

In this model the word problem is treated as a situation which consists of a question and some important information. The expected role of the student is to identify the question and the important data and then to select the mathematical operation/s for solving the problem. The special emphasis that is given in the selection of the requisite operation is also proven by the fact that in the grade-six textbook there is a separate unit with the title 'We find the operations we need to perform for solving the problem' (Ministry of National Education and Religion, 1993b, v. 1, p. 12). Particularly in the teacher's maths guide the objectives for this lesson are stated as follows:

'The students must practise in order:

- to identify the words or the phrases that determine the operations they need to perform in order to solve a problem.

- to thoroughly examine the procedure and the function of the operations in a problem' (Ministry of National Education and Religion, 1994, p. 17).

According to this model the word problem, instead of being viewed as a description of a situation requiring mathematical analysis, is viewed as a set of words or phrases, only some of which are important in deciding what to do to obtain the solution. Neither uncertainty about how to proceed on the part of the student nor a desire to solve the problem are necessary components. On the contrary, it is assumed that the student may be interested in the problem and the requisite operation may be immediately obvious to him/her. Inherent in this model is also the assumption that the mathematics is 'in' the problem, waiting for the student to 'detect' or 'uncover' it. In other words the emphasis on the solution process indicates that the word problems exist independently of the student.

I have noticed that in most maths units there is a concentration of many and different word problems, and that their selection is not based on any obvious pedagogical objectives. I think it is impossible for students to solve these problems in class, within the limited time provided by the curriculum. The great number of problems and their degree of difficulty may threaten both the students and their parents. As an unpleasant consequence the students may develop a tendency to dislike word problems.

2.7 Word problems in the teachers' maths guides

Word problems are incorporated in the lower elementary grades and have begun to appear more frequently in the curriculum of grades four, five and six. It is important to note that from the school year 1993-94 the maths textbooks for the primary grades four, five and six are revised editions. In the instructions of the Pedagogical Institute (Teachers' guides for grade-four, five and six) it is said that this revised edition has introduced some changes in the textbooks. One of the most important of these is the emphasis on problem solving. Interestingly enough, despite mention of problem solving, there is no definition provided. In general the structure of the revised textbooks is based mainly on the following principles:

(1) The topic of each unit appears in problem or question form and the students using their previous knowledge and experiences try to approach it.
(2) The students 'think' of how to solve the problem. In other words they look for the known and the unknown elements of the problem, they find the interrelationships between them, they devise a plan of action and then they solve the problem.
(3) The students 'think further', that is, they also examine other aspects of the problem:
    -they solve the problem using other methods,
    -they transform the original problem and they form new ones (e.g. inverse problems),
    -they state similar problems and solve them.
(4) The practical application of knowledge is sought through problem solving.

2.8 Reasons for choosing grade-four

In this section I explain the reasons why I chose to work with grade-four. Examining the Greek mathematics curriculum for all the primary grades (1 - 6) I found that although students encounter exercises (in a problem form) from grade-one, they actually start to be taught problems in a more thorough way from grade-four. According to the curriculum, in this grade, they begin first to solve the problem by structuring the relationships involved in a mathematical expression, second to find other ways of solving the original problem, third to create another problem with the same numbers but a different context, and finally to solve the original problem with different numbers.

In addition, the maths textbooks used in the second and third grades, include very straightforward problems. I have noticed that at the end of each section there is a number of word problems requiring specific arithmetic operations, such as multiplying two numbers or dividing two numbers. When students know the section is on multiplication or division (i.e. from the lesson's title), they will often multiply or divide without reading and conceptualising the problem.

2.9 Reasons for choosing specific units on word problems

In this section I explain why I chose to work on some specific units of word problems with grade-four students. First the units I chose to experiment with provided limited indication of the operations required to solve the problems. The students were accustomed to being informed which operation to use for solving the problems usually from the unit's title (e.g. 'From division to multiplication').

However, the titles of the units I chose, like 'How we solve problems using the four operations' or 'Problems with more unknowns' did not provide any indication (hint) to the students which operations they should use for solving them. Although the title may have implied that the following problems could have been solved by using one or more of the four basic operations, it did not indicate a specific operation. It was left to the student to decide which operations to use and why. Examples of such problems are:

Unit title: 'How we solve problems of four operations'
'A pedestrian walks at a speed of 4 km per hour. The truck's speed is 14 times the speed of the pedestrian. The speed of the aeroplane is 8 times the speed of the truck.
What is the aeroplane's speed? (Ministry of National Education and Religion, 1993a, p. 57).

Unit title: 'Problems with more unknowns'

'A truck will transport 865 sacks of cement. Each sack weights 50 kilos. The truck has a loading capacity of 87 sacks.

How many journeys must it make with full load ?

How many sacks will it transport in the last journey ?

How much will a full load of cement weigh ? (Ministry of National Education and Religion, 1993a, p. 62).

It was the first time that grade-four students had encountered problems of this kind, where the title of the lesson gave them no guide. They did not know immediately which procedure to follow or which technique to use in order to resolve the problem. In other words the approach to the solution was not provided for them, necessitating them to analyse the problem before they could solve it. These were the kinds of word problems used in my teaching experiments.

Secondly, these word problems were different from what the grade-four students had been accustomed to, in the sense that their solution required more than one operation. The grade-four students were only familiar with the one-step problems from their textbook, but they had no experience in solving multi-step problems.

Taking into account that the title of the unit would be of no help to the students, in conjunction with the fact that word problems would require more than one-step, I felt that this would provide the students with a new experience, which would be worth exploring in a group work situation.

2.10 The pilot study

My pilot study took place during March - April 1994 in the grade-four class of the Greek primary school. The teacher was about my age and she was also a research student in the field of sociology. When I asked her if I could make some observations in her maths sessions she had no objection. She appeared to be very cooperative and willing to help me in anyway she could.

During my pilot study I explored different aspects of her teaching and I started to gain experience in conducting research. The first day I attended the lesson without keeping field-notes or using a tape-recorder. The next time I asked the teacher if I could take notes while she was teaching (problem solving) and she raised no objection. So I spent the next four lessons making observations on the fourth-grade class during math sessions on word problems and recording these in field-notes. At the end of each math session I discussed with the teacher different points from the lesson and we commented on the students' behaviour.

In our discussions the teacher appeared to be frank and unbiased. She did not have any difficulty in commenting on anything I asked her about the unit, the students' behaviour or even her teaching actions. Having self-confidence, she expressed her opinions and tried to support them with valid arguments.

After the fifth observation, with the teacher's permission, I started to use the tape-recorder in order to have a better and permanent picture of what went on during the maths session. As the tape-recorder was switched on I kept notes of the context in the class as well as of other incidents which were not captured by the recorder. I discussed also with the teacher the possibility of exploring (together) some points with her students in class. She agreed to my suggestion and then we spent five sessions basically exploring two things: (1) placing the students to work in groups and (2) asking them to justify their solution methods.

(1) Working in groups: The teacher divided the class in four small groups with four members in each group. I did not take any initiative in forming these but left that to the teacher's discretion, for two reasons. First because the teacher knew her students better than I and second, since she was doing the actual teaching I did not want to interfere by imposing my own selections. Although boys and

girls were placed in each group, no provision for mixed ability groups was taken into consideration. Observing the groups I got the impression that in one of them were placed the high-attainers in the class while the other three groups consisted mainly of the medium and low-attainers.

The students working in groups on the word problems were given the opportunity to listen to each other, to argue about the solution approach, to take responsibility. Overall they appeared to prefer the way of working in groups. I noticed that every time they saw me coming in their class they ran to arrange the tables. In some cases the students produced more solutions than expected. This happened basically, with the group of the high-attainers. In contrast the groups with the medium and low-attainers sometimes were successful in finding a solution and sometimes were not.

(2) Students justifying their solution methods: This objective did not prove to be easy to accomplish. The students were specifying the necessary operations for the word problems but they were unable to give reasons for their choices.

At the end of my pilot study I interviewed the students asking them questions about their experiences from working in groups. From the comments they made, they seemed to prefer the group work. Characteristically, two students commented:

S1: Working individually takes more time and we do not learn anything.
S2: When we work in groups we solve the problems faster and easier.

Certainly, my experiences from this pilot study and the several issues which were raised, helped me to think how I would be able to organise better the main study. Some implications of the pilot study for the main one were:

- to observe first the class with which I was going to work
- to cooperate with the teacher

- to define criteria for establishing mixed ability groups
- to find a way of addressing the second goal of the pilot study (the 'why' question)
- to use pre- and post-tests and pre- and post-interviews
- to arrange the tables in groups.

2.11 My original intentions

My original plans were to work with the regular teacher of the fourth-grade class. The afore-mentioned teacher was not the one involved in the pilot study which had taken place the year before with a different teacher. For this reason I arranged meetings with the teacher in which I explained both the theoretical framework of my research and what I would like us to explore together in class. I tried to be sincere, specific and analytic to the teacher as far as was possible. In addition I said to the teacher that after each session she could have my field-notes to read, as I had already done during my preliminary observations of her class.

In the beginning the teacher seemed to accept my plan and showed enthusiasm for it. After the fifth meeting with her, it was time to start the intervention, but there was always an unexpected problem preventing us from starting the experiment. One day when we had arranged to start the teaching experiment the teacher told me that it was not possible as the students had to write a test and on another occasion, when we had agreed to start, she was absent.

After the successive postponements and the delay in starting the teaching experiment I felt that something was going wrong. Discussing with the teacher I realised that she was worried about what we had set up to explore together. One day she asked me if I would like to do the teaching myself. Although I discussed the matter with her she insisted on being an observer. Her attitude deteriorated even further and finally she made clear to me that she was not going to engage in the teaching process at all. Then I decided to do the teaching myself undertaking both roles, that of the teacher and that of the researcher, as this class was the only grade-four class in the school and I therefore had no alternative. Hence, the teacher's behaviour was influential in determining aspects of the methodology of this research.

2.12 Refinements of the methodological approach

Partly as a result of the constraints involved in the study, the design changed to some extent. The end result was that the case study had some aspects of (1) a teaching experiment approach and (2) an action research approach.

(1) The teaching experiment approach

As the aim of my research was to explore changes in attitudes and behaviour of primary pupils when they move from traditional class teaching to group work, I used a methodology which had aspects of a 'teaching experiment' in the sense that: 'a teaching strategy is developed that involves systematic intervention and stimulation of the students' learning, and both the effectiveness of the teaching strategy and the reasons for its effectiveness are determined' (Romberg, 1992, p. 57).

The (constructivist) teaching experiment is a research methodology (Steffe, 1983) which has been employed by Steffe and his colleagues for more than a decade. It is characterised by the researcher acting as a teacher, determining the students' mathematical knowledge at a given time, and, ultimately, creating a model of learning specific content. 'The basic unrelenting goal of a teaching experiment is for the researcher to learn the mathematical knowledge of the involved children and how they construct it' (Steffe, 1991, 178).

According to Cobb and Steffe (1983) teaching experiments (constructivist and nonconstructivist) share three general characteristics: (i) the 'long-term' interaction between the experimenter and a group of children, (ii) the process of a dynamic passage from one state of knowledge to another and (iii) the data are generally qualitative rather than quantitative.

The qualitative data is obtained from two possible sources. The first source is teaching episodes with the children and the second source is clinical interviews conducted at some points during the teaching experiment.

Menchinskaya (1969) reviewing instructional psychology in Soviet literature identifies two types of teaching experiments. She calls the first type of teaching experiment a *macroscheme*: 'changes are studied in a pupil's school activity and development as he makes the transition from one age level to another, from one level of instruction to another' (p. 5, cited in Cobb and Steffe, 1983, p. 87).

This methodology was used by Cobb and his colleagues (Cobb et al., 1991, Yackel et al., 1991) to conduct a constructivist classroom teaching experiment in a Grade 2 classroom to both develop the instructional activities and investigate the children's learning.

In order for the teacher to gain insights into the children's mathematical thinking, it was essential that the children accept the obligation to explain their mathematical activity. To bring this about, the teacher had to initiate and guide the negotiation of social norms for classroom activity. The teacher used two strategies to initiate and guide the construction of these obligations and expectations. First, she framed situations that arose spontaneously as paradigm cases and initiated class discussions of obligations and expectations with respect to these cases. Second, she initiated class discussions of the children's obligations prior to small group work (Yackel et al., 1991).

Since the classroom teaching experiment did not use an external reward structure, several quantitative assessments (Nicholls et al., 1990) were designed to analyse the goals, beliefs and motivations developed by children in the project classroom and to compare them with those of other second graders in the same school. The results of these assessments are that project students have higher levels of conceptual understanding in arithmetic, hold stronger beliefs about the importance of understanding and collaborating, and attribute less importance to conforming to the solution methods of others.

The second type of teaching experiment identified by Menchinskaya is the *microscheme*, where 'in a single pupil the transition is observed from ignorance to knowledge, from a less perfect mode of school work to a more perfect one' (Menchinskaya, 1969, p. 6, cited in Cobb and Steffe, 1983, p. 87).

A teaching experiment of this type was carried out by Cobb and Steffe (1983). Specifically they conducted the one-to-one constructivist teaching experiment, which is an extension of Piaget's clinical interview methodology, to investigate the process by which a single child constructs mathematical knowledge. When interacting with a child, the researcher focuses on what the child might be thinking and uses these interpretations of the child's activity to formulate further tasks to pose. The tentative models which the teacher / researcher develops by intensively analysing the child's behaviour, capture his/her knowledge of recurrent features of the activity of doing mathematics and they also embody his/her suggestions of how to aid the child to construct mathematical knowledge (Cobb and Steffe, 1983).

However, the microscheme tends to emphasise the cognitions of individual children at the expense of social interaction. In the course of the one-to-one teaching experiment the researcher focuses almost exclusively on what the child might be thinking and implicitly takes the social process of the interview for granted.

Work similar to Cobb's but much shorter in time scale has been carried out by Hart (1984), Booth (1984) and Johnson (Ed.) (1989). Some pilots taught by the researchers, cognitive growth, small groups, pre-tests and post-tests, children's methods, have been some of the characteristics of these studies.

Implications for my research

Although my research has some of the characteristics of the teaching experiment as defined by Romberg (1992), Menchinskaya (1969) and outlined by Cobb and Steffe (1983), it does not represent a teaching experiment methodology in the full sense. The reasons for this is that my teaching experiment does not have the 'systematic intervention' or the 'long-term' interaction between the experimenter and a group of children, as it was required by Romberg and Cobb & Steffe respectively. It is actually closest to Hart (1984), Booth (1984) and Johnson (Ed.) (1989), but while their research was limited to cognitive change, my research concentrates more on changes in attitudes and behaviour.

Another difference between my approach and that of Cobb and his colleagues (the classroom teaching experiment) is that in my case the instructional materials (word problems) were given in the (grade's four) set textbook, whereas Cobb and his colleagues develop instructional activities on an ongoing basis by drawing on daily analyses of children's mathematical activity.

Creating a new working scheme for students - group work - required a teaching strategy different from the traditional strategies. So, in order to study the effectiveness of this strategy (group work) on students' attitudes and behaviour, as well as the reasons for its effectiveness, I conducted a teaching experiment which consisted of four stages:

(i) preliminary observations
(ii) pre-test and pre-interview
(iii) actual teaching
(iv) post-test and post-interview

Two teaching experiments (TE-1, TE-2) were conducted. The second included some refinements of the method used in the first. Because the details of the design were slightly different in the two experiments, these details will be described in Chapters 3 and 4.

(2) The action research approach

Due to the constraints involved in the study, I set out to establish a piece of research which also includes some aspects of action research (Lampert, 1990). Characteristic of this kind of research is that the teacher assumes another role, that of the researcher. 'The purpose of action research is not to derive new theories that can be applied to reform practice, but to subject theory to the conditions of practice and examine practical action in a concrete situation so that theory and practice develop interactively' (Lampert, 1990, p. 36).

Elliot (1992) defines action research as 'the study of a social situation with a view to improving the quality of the action within it' (p. 69). Action research as defined by Elliot allows a wide range of approaches and methods, while stressing the guiding ideal of a clear link between theory and practice, and hence between the role of the practitioner and the researcher. It involves reflection on practice in order to evaluate, develop, or change this practice as a result.

Carr and Kemmis (1986) are more explicit about aims:

> 'There are two essential aims of all action research: to *improve* and to *involve*. Action research aims at improvement in three areas: firstly, the improvement of *practice*; secondly, the improvement of the *understanding* of the practice by its practitioners; and thirdly, the improvement of the *situation* in which the practice takes place. The aim of *involvement* stands shoulder to shoulder with the aim of *improvement*' (p. 165).

In relation to the practice of action research they add:

> 'It can be argued that three conditions are individually necessary and jointly sufficiently for action research to be said to exist: firstly, a project takes as its subject-matter a social practice, regarding it as a form of strategic action susceptible of improvement; secondly, the project proceeds through a spiral of cycles planning, acting, observing and reflecting, with each of these activities being systematically and self-critically implemented and interrelated; thirdly, the project involves those responsible for the practice in each of the moments of the activity, widening participation in the project gradually to include others affected by the practice, and maintaining collaborative control of the process' (p. 165-166).

In this study I have attempted to draw on some aspects of action research by incorporating into my teaching method a cycle that allowed flexibility in the way in which I was to operate. As a result of what I gathered from one session of teaching and learning, I used this information to modify the next stage.

Following an action research framework in a case study, I found myself to be involved in a continuing process of reflection on the meanings of the students' behaviour. Students' behaviours were open to interpretation and re-interpretation through reflective practice. Teaching word problem solving was viewed to be grounded in reflective experiences of concrete situations. Making comparisons with past experiences illuminated practical features of the present situation.

I found improvement of practice in cooperative work and classroom discussion, primarily to be based on my own capacity for discrimination and interpretation through reflective practice. During the teaching experiment, judgements on practical problems and action hypotheses about strategies for resolving them, were reflectively tested and evaluated. I viewed the process of testing hypotheses to be tied in with the process of evaluating teaching. In fact, evaluating was an integral part in my approach.

Conducting a case study research with some aspects of a teaching experiment and some aspects of action research required that the teaching method I used could not be rigid. It was very important to build flexibility into my method, because it allowed me the opportunity to learn from our (teacher and pupil) experiences, and to modify further lessons accordingly.

As I became teacher / researcher my research methodology adopted features of action research, but since I was not the regular teacher, it was not an action research in the full sense.

Therefore, I would argue that my research is best characterised as a mixed methodological approach. It is a case study with some of the characteristics of teaching experiment and it also has some of the characteristics of action research.

2.13 Data collection

According to Adelman et al. (1980) 'case study uses a variety of techniques, often 'sociological' in character' (p. 54). I believe a scientific model of research which involves only the systematic collection of quantitative data in my situation, with fifteen children in TE-1 and six children in TE-2, with the teacher / researcher as the only observer, would limit the amount of data on which quantitative explanations and theories could be based. On the contrary, a qualitative approach would produce empirical data of rich and analytical descriptions which would interpret more effectively students' attitudes and behaviour.

I was interested in changes of the students' attitudes and behaviour when they move from traditional class teaching to group work on word problem solving.

Indeed I found it difficult to reconcile the hard and often mechanistic view of quantitative research, with the group of individuals in these two classes where they exercised choice and expressed their own individuality in many different ways. To look objectively at these children's actions, as unproblematical, self evident and quantifiable, would be to greatly underestimate them as individuals and take much away from the insightful and reflective nature of being a teacher / researcher. Being objective and ensuring a clinical detachment from the subjects to avoid personal bias, would be difficult although some objectivity needed to be maintained. Thus I chose mainly a qualitative research approach to understand the students' meanings of actions and events, rather than simply judging them numerically.

Thus, I basically adopted an interpretative approach to my research and started from the particular circumstances of the individuals within the context of the two grade-four classrooms. However, in addition to the thick descriptions of the students' attitudes and behaviour, I used a quantitative approach at the beginning and at the end of the teaching experiments.

The quantitative approach was used only for evaluating students' learning (in a more formal manner). It was restricted only to a pre-written test, a post-written test, and (only in TE-2) a pre-questionnaire and a post-questionnaire.

By keeping a research / field-work diary I hoped to capture some of the fluidity, spontaneity and creativity of classroom life. Interpretative techniques deliberately reduce the distance between researcher and subject, allowing me, as a classroom teacher, to carry out the research myself, simultaneously remaining both teacher and researcher. It allows for the researcher to become involved, yet at the same time remaining detached to make observations. Participant observation (Spradley, 1980) and reflection were important to the success of this study and were central to my methodology.

In my research the means of collecting data include:

- tape-recording (class and group discussion)
- field-work notes
- test (written pre-test, written post-test)
- interview (pre-interview, post-interview)
- questionnaire (pre-questionnaire, post-questionnaire)
- observation

The methods of collecting data in each teaching experiment (TE-1, TE-2) are shown in the following Figure (Figure 2.1).

Figure 2.1 Methods of collecting data

TEACHING EXPERIMENT 1	TEACHING EXPERIMENT 2
Observation of the regular teacher	Observation of the regular teacher
Interview with the regular teacher	Interview with the regular teacher
Pre-tests	Pre-tests
Pre-interviews	Pre-interviews
	Pre-questionnaires
Tape-recording of the lessons	Tape-recording of the lessons
Diary entries	Diary entries
	Students' work sheets
Post-tests	Post-tests
Post-interviews	Post-interviews
	Post-questionnaires

Since my research was based on a case study, I wanted multiple sources of data. A particular technique for validating findings was triangulation. So the multiple sources of data collected from interview observations, class observations and written data from the questionnaires provided some degree of triangulation and helped me to validate the findings. Woods (1991) points out that 'the major means, however of validating accounts and observation, ... is through 'triangulation'. Triangles possess enormous strength' (p. 87). Examples of triangulation occur especially in Chapter 4, sections 4.5.1 and 4.5.2.

In the following, I describe the first two means of collecting data in my research (i.e. tape-recording, research / field-work diary), while the design of the last four methods is described in Chapter 3 and Chapter 4 since it differed between the two experiments.

(1a) Classroom recording in teaching experiment-1

I had initially hoped that I could record whole group discussions by using a tape-recorder placed on a table, but an initial trial run had shown that background noise and group movements had made the conversations inaudible. I also observed that it was inhibiting group talk to a greater extent than I initially thought it would, even after the children had become used to the tape-recorder being in the classroom.

Eventually I decided to use the tape-recorder to record whole class discussions between the teacher and the entire group. This provided a means of recording the ideas of both individuals and whole groups and inter group discussion when and as it occurred. The presence of the tape-recorder in class did not seem to affect the group members in their explanations, except the first two sessions that were necessary for them in order to be accustomed to this new facility. I should emphasise that three times the tapes of the discussions were available for children themselves to listen. The children welcomed this, and appeared to derive pleasure from it. I did this because I wanted to make working with groups a joint venture rather than something that was imposed upon them. All of this seemed to help to make my experiment something that was shared between teacher / researcher and the children.

The tape-recorder was used throughout my teaching experiment. During the actual teaching stage of my research I taught grade-four class ten units

(Appendix 1) in sixteen maths sessions and all of them were tape-recorded and transcribed. I found transcription of the tapes a difficult and time-consuming task. The major difficulties lay in (1) transcribing utterances that were spoken simultaneously with other utterances and (2) attributing utterances to speakers, as the children's voices often sounded very alike on tape. I should point out that, although the transcription time was varied according to particular features of different sessions, the average time for transcribing a tape of a session (45 minutes) was from four to six hours. Running the tape recorder over and over again and referring to the plan with the groups, I managed to transcribe all the tapes.

Another constraint of this research concerns the kind of word problems used for learning activities. As I already mentioned, in all sessions of the teaching experiment-1 I used the textbook's word problems as the regular classroom teacher and the students' parents were very concerned with covering the syllabus for grade-four. The students seemed to prefer the textbook's problems as some of them could come to the lesson prepared. The pre-test and the post-test were the only two cases where the students met word problems that they had not seen before (in the sense that they were unknown to them).

(1b) Classroom recording in teaching experiment-2

The tape-recorder was also used in teaching experiment-2. During the actual teaching stage in TE-2, I taught grade-four class four units (Appendix 2) in ten maths sessions.

In designing the teaching experiment-2, I had thought to avoid visiting groups, as my presence there could have changed the interactive balance. After the first session, this proved to be unavoidable, as the students were identifying operations without making it clear which numbers they were referring to. So during cooperative group work, part of the time I observed the first group and for part of the time I observed the second group.

Another difficulty involved the use of the tape-recorders because the class was small; each group's movements and conversations were recorded on each other's tape-recorder. This made transcribing the tapes an extremely difficult task. Furthermore in some cases it was impossible to attribute student responses that were made simultaneously with responses from other students, as the children's voices often sounded very alike on the tape.

(2) The use of the research / field-work diary

Prior to looking at the groups engaged in word problem solving activities, I decided to observe the classes in mathematics sessions with their regular teachers, so that I would have an idea of what was actually taking place. At the same time I could gain experience in observing and tape-recording, in addition to my previous experience in the pilot study. I wanted to establish a workable method that would enable me to make observations that were both valid and reliable as true reflections of what was actually happening.

From my preliminary observations I felt that it would be possible to make general notes about the degree to which individuals were participating in the group work. However, I found it almost impossible at the beginning of the actual teaching (in TE-1) to observe a particular group for an extended period of time or in any systematic manner, because of interruptions by other groups and the need to encourage individuals and sort out minor disputes as and when they occurred. But as time went on, and as collaboration improved and groups became more independent, more time became available.

Wolcott (1981) outlines a strategy for observing nothing in particular, that when something unusual happens it will jump out from the drab background and force itself onto the observer. The metaphor he uses is that of the observer as a radar scan, watching for the blip that signals a disturbance. In many ways there was an element of this in my diary observations.

During the actual teaching, as well as before and after, I used a research / field-work diary for my private communications. There were two particular purposes for which the diary was especially useful for me. One in terms of the research itself and the other in terms of my own involvement in the research. The diary was a valuable document which included observations of students, descriptions of their interrelationships, key points in the teaching experiment, interesting classroom events, dates, reactions, views, and feelings. Reading the diary, it was simple for me to recollect past events and interpret them. It helped me to become sceptical and to seek to understand the interpretative frame behind all the records included. It also proved an important complement for my transcriptions in terms of the classroom context that was not captured by the tape-recorder. The second important use of the diary was in monitoring my own involvement in the research. Thus it included details on how my teaching experiment was initially conceived, how it was related to my own personal development, the problems involved in conducting the experiment, fears, worries and so on.

2.14 Common constraints in both experiments

As a teacher / researcher there were constraints to the way I was to approach this piece of classroom research. Time was a major factor, and creating time to actually observe what the children were doing, listen to their explanations, and manage whole class discussions, proved to be extremely difficult.

It is important to note that I was not the regular classroom teacher. As a result the teaching experiment approach and the action research method I used were not conducted in my own classroom setting. Therefore, I had only limited time in any given week in which to carry out this research. The students were exposed to this approach only during the teaching experiment. More time could possibly have allowed me to extend the experimental approach to other areas of the curriculum and compare the results.

Another consequence of the limited time available was associated with the difficulty of covering the maths syllabus. In fact, in three of the sixteen sessions I taught in TE-1, I worked with students on two problems only each time, thus there was a sense of falling behind in the mathematical content. I found that working in a different classroom setting put a lot of pressure on me as I had to carry out the research at my own pace and at the same time to cover the mathematical syllabus.

Finally, one of the major constraints of being teacher / researcher, as previously mentioned, was being able to observe and at the same time manage the class. I found it very difficult to observe the groups in any systematic manner but I believe that I have been able to succeed sufficiently in deriving valid data.

2.15 Timeline of the research

The timeline of the research is shown in the following Figure (Figure 2.2).

Figure 2.2 The timeline of the research

School year 1993-94 :	**Pilot study**	
March - April 94	**Observations / Explorations**	**10 sessions**
School year 1994-95 :	**Teaching experiment-1**	
December 94	**Preliminary observations**	**3 sessions**
January - March 95	**Preliminary observations**	**9 sessions**
May 95	**Pre-test / Pre-interview**	
May 95 - June 95	**Actual teaching**	**16 sessions**
June 95	**Post-test / Post-interview**	
School year 1997-98 :	**Teaching experiment-2**	
April 98	**Preliminary observations**	**6 sessions**
April 98	**Pre-test / Pre-interview**	
	Pre-questionnaire	
May 98	**Actual teaching**	**10 sessions**
June 98	**Post-test / Post-interview**	
	Post-questionnaire	

2.16 Analysis of data

The analysis of the tests, questionnaires and their accompanied interviews are discussed in separate sections in Chapters 3 and 4 (see sections 3.5, 4.5).

(1) The emergence of the categories

First of all I should make clear that all the data collected in this study was in the Greek language, therefore the analysis took place in Greek. There were two main reasons for this: Firstly it was easier for me to conduct the analysis in my native language, and secondly because it would save me the time of translating into English teaching episodes and diary entries which I would not need them in the discussion of findings. So, after I finished the analysis, I translated into English only the teaching episodes and the diary entries that I would use to support my arguments.

As I have already explained, different methods of data collection were combined in my research. However, the main source of data during the actual teaching stage of the experiment were tape-recordings and diary entries.

The method used for analysing this data is the 'constant comparative method' as it has been outlined by Glaser (1978) as well as by Lincoln and Guba (1985).

I started the analysis of the transcribed tapes by coding the teaching episodes and classifying them in categories. Coding a teaching episode for a category, I had to compare it with the previous episodes coded in the same category. Although at the beginning I worked with cards containing summaries of episodes, very early on I found that working with the episodes themselves was more helpful.

Thus, photocopying the episodes of each lesson and separating them by cutting them into pieces, helped me to classify them into categories, because I could shuffle them around and see where they fitted best. The criterion for classifying an episode into a specific category was in accordance to the information included in each episode. However in some cases an episode had to be classified into more categories, due to the fact that it included information relevant to these categories.

Directly related to the transcripts were my diary entries which seemed to have an important role in triangulation. Diary entries proved supportive but also complementary to the transcripts as they often provided information about the classroom context that was not captured by the tape-recorder. Students' actions, reactions and feelings expressed by nods and gestures ('body language'), as well as personal thoughts and impressions were recorded in my diary. However, in my effort to explain the students' attitude and behaviour in group work, in a better way, transcripts and diary entries were treated and woven together.

The writing of memos helped me to uncover the properties of the category. Multiple memos which were written about a category provided a useful description of it. I found it helped me to define the category by using a separate card for each category and writing the memos on it.

At this point however I must make clear that although the categories emerged from the collected data (and therefore they were not pre-determined), as is explained below, my own knowledge and belief and particularly my acquaintance with the work of Cobb, is likely to have influenced the outcome.

Each category was the result of a number of episodes with the same theme grouped together. Finally there were 12 categories for TE-1 (omitting the "miscellaneous" category) and 11 for TE-2. These are listed in the next section.

The next step in the analysis was to finally check that the teaching episodes classified in a category matched the description of this category.

(2) From the categories to the core categories

The process of exposing episodes and categories to searching criticism enabled me then to integrate the categories. The relations discovered among categories and their properties led me to the core categories which were selected for their (concentrated) explanatory power. Three core categories in TE-1 as well as in TE-2 appeared to explain the students' attitude and behaviour during group work on word problem solving.

The original integration of the 13 categories in TE-1 revealed seven core categories: (i) "students' way of working", (ii) "students' attitudes to word problems", (iii) "the teacher's role", (iv) "establishing norms", (v) "students' self-confidence", (vi) "students' commenting on my teaching approach", (vii) "miscellaneous".

However, at the end, after comparisons and revisions I came up with only three core categories. That is, three of the original categories were integrated and one of them was left out.

The category "establishing norms" was integrated with the first core category "students' way of working". I found that students' responding following the class norms seemed to be closely related to the theme of the first core category. Originally, I thought that the category "establishing norms" would relate to the core category "teacher's role" because the establishment of norms would be directly affected by the teacher's actions. However, after a closer analysis I found that the category would be better integrated with "students way of working". The reason for this being that I put more value on the students' behaviour (i.e. responses) rather than on the teacher's action. At the same time however the teacher's contribution was acknowledged as important.

The original categories "students' self-confidence" and "students' commenting on my teaching approach", both were grouped (incorporated) with the other categories which resulted to the core category "students' attitudes to word problems". The reason for the integration of the category "students' self-confidence" is that its theme is expressed by the core category "students' attitudes to word problems", as it simply reveals how two low-attainers built up their self-confidence during group work on word problem solving. The category "students' commenting on my teaching approach" was the only category with a small number of episodes, and the reason for its integration to the core category "students' attitudes to word problems" was that the students' responses reflected their attitudes towards word problem solving.

The "miscellaneous" category included all the isolated episodes (i.e. those who were not classified in any category and basically contained procedural information) as well as information related to the students' behaviour before starting the maths lesson (i.e. arranging the tables, setting up the equipment etc.). I therefore left this category out, as it did not related to the students' attitudes and behaviour I was particularly interested.

The categories and core categories which emerged in TE-1 were:

Students' way of working

- Students' demonstrating their collaboration
- Students revising their thinking
- Students incorporating other students' solutions
- Students developing arguments to support their solution methods
- Establishing norms

Students' attitudes to word problems

- Students' approaches to word problems
- Students' interpretation of word problems
- Students' self-confidence

- Students commenting on my teaching approach

The teacher's role

- Management of group task
- Class discussions
- The value of building a framework of communication

Although the categories, and therefore their core categories, emerged from the data, I have to explain that in the analysis of data in TE-1 most of the emphasis was placed on classroom discussion (between the groups and the teacher), while in TE-2 there was a shift in the emphasis towards group discussion (between the group members).

The original integration of the 11 categories in TE-2 revealed four core categories: (i) "students' attitudes to word problems", (ii) "students' collaboration during group work", (iii) "the group's influence on students' behaviour", (iv) "a special case" (Argyris' behaviour).

But since the last category was not a core category in the true sense, it was integrated with the third core category. The reason for this integration was that at the beginning Argyris' disruptive behaviour could be seen as a result of the group work, while later group work helped him again to solve his behaviour problem. In other words group work had certainly an influence on Argyris' behaviour. Thus, Argyris' case, although exceptional, was incorporated with the core category "the group's influence on students' behaviour".

The categories and core categories which emerged in TE-2 were:

Students' attitudes to word problems during group work

- Developing meaningful solutions versus producing answers
- Verifying the answer versus evaluating the reasonableness of the answer
- Students identifying special roles for themselves

- Writing out the 'thinking procedure' and the explanation of the 'answer'

Students' collaboration during group work

- Students' effort to keep the group work going
- Leaders with negotiating behaviour
- Students' interdependence

The group's influence on students' behaviour

- Building up students' self-confidence
- Communicating one's understanding leads to strengthening that understanding
- Students improving their behaviour
- A special case

(3) The criteria for choosing particular illustrative episodes in Chapters 3 and 4

The criteria for selecting particular illustrative episodes to the categories were the following:

(i) their clarity for presentation: I wanted the episode to explain the category's theme as clear as possible.
(ii) their brevity: I tried to select brief episodes, even though this was not always possible in the presentation of group work or inter group discussion.
(iii) their singularity: I wanted the episode I used to illustrate a single category and therefore not to be reused to illustrate another category.
(iv) variety of groups: I tried where it was possible to use as illustrative episodes, extracts which covered all the groups in my class.

Some examples of how specific episodes relate to categories are cited at the end of the section.

At this point I should make it clear that although I selected episodes to explain the category's theme as clearly as possible, this does not imply that the behaviour exhibited in the episode was demonstrated by all groups or group members to the same degree.

Most of the groups appeared to satisfy the attitude or behaviour described in the category, in one way or another, but certainly, in some episodes, there were groups who had limited participation. Moreover, a behaviour which was demonstrated by a group member does not imply that necessarily this behaviour could be attributed to all members of this group.

I thought it was necessary to make this clarification as it may not always be clearly spelt out in the discussion of the findings in Chapter 3 and Chapter 4.

(4) The basis of the conclusions stated in Chapter 5

My overall conclusions were based upon the contents of the categories and the core categories which emerged from both experiments, as well as on the combination and comparison of these categories.

EXAMPLE 1 "Students' interpretation of word problems"
(Part of core category "students' attitudes to word problems")

The following extract shows the coding for a section of a teaching episode in which a group of students solved the problem in a way that was more meaningful to them. The students treated the situation as if it were a real life situation and they identified themselves with it. The reason I chose to use this episode among others to demonstrate this category was because of the 'insightful' solution presented as a coordinated effort by all group members.

Teaching episode	**Information included**
I : Okay, you made it clear now. Now let's hear and the last group to see how they approached the solution. Can you start Elias?	**Encouraging participation**
E: Sir we followed a different way. We thought about each day separately. So for the first day we subtracted our expenditures from our collections and we found our net income to be 21.290,5 drachmas. For the second day we subtracted again our expenditures from our collections and we found our net income to be 31.768,5 drachmas and for the third day we found that our net income was 31.659,5 drachmas. Then we added together the net income from each day and we found that our total net income was 84.718,5 drachmas.	**4th solution (a different insight to the problem)**
T : [from the same group, G3] : Our best day was the second. I : Why? K : [From another group, G1] : Since it was the day that the shop had the best net income. T : Because sir, the second day we had the most profit.	**subsidiary information (not asked for in problem)**
...	
E: It made more sense to us to find the net income from each day separately.	**Effect of context**
T: If we were the shopkeepers everyday we had to know our net income ... [pause] our profit.	
Y: [from another group, G1] : Your way is fine.	**Reward by another group**

EXAMPLE 2 "Establishing norms"

(Part of core category "students' way of working")

The following three episodes were among the others which had been classified in the category "establishing norms". The phrases in italic are the key sections of the episode.

Episode 1

When two of the groups had not made any progress in forming an addition of decimal numbers correctly, I wrote on the board

```
   38,600
 + 112,5
 ---------
```

and I asked for the students' attention. The following is an extract from the discussion that developed mainly with students from the groups that had formed the addition correctly:

A : *It' s wrong ... the decimal point.*
I : *Remember not to use words like "wrong", "mistake" etc.*
A : *Sorry sir. We don' t agree with the decimal point. It must be placed at six.*
I : Can you tell us why?

Episode 2

When the students' found a point of dispute at a subtraction of decimal numbers their argument went as follows (see also Example 3):

I : Yiannis can you come to the board to execute the subtraction?
Y : Okay, sir.

Yiannis comes to the board to execute the subtraction (on decimal numbers). The student executes it as follows:

```
  179.983,50
-  95.265,00
 ------------
  084.718,50
```

I : [I read] : 84.718,50. What is ...

M: [She interrupts] : *Sir, I don' t agree, he should not have put a zero in front of 84.*

I : How does anyone else feel?

Y : It does not matter.

I : Can you explain to us why "it does not matter?"

Y : The number does not change.

M: *No sir, our teacher has told us not to put a zero there.*

I : What can we put instead?

F : An equal sign.

L : *Maybe he did not know it.*

I : Let's look at the two numbers the one with zero in front and the one with an equal sign in front.
[I wrote them on the board.]

Episode 3

From the list of the episodes which had been classified in the category "establishing norms", I selected the following one, as I though that it explains clearly the category's theme.

I : What do you think you will use here?

G: Subtraction.

I : Can you explain to us why?

G: In order to find his profit.

I : What will you subtract?

G: The income from the expenses, the expenses from the income. 350.000 drachmas are the expenses and 60.000 drachmas are the income. I will subtract 350.000 from 60.000.
[The actual figures were 350.000 for the expenses and 600.000 for the income.]

L : Sir?

I : Yes, Litsa.

L : *I do not exactly agree with Georgia. Not with the operation but how she said it. She said 350.000 from 60.000. Therefore either she forgot to put the zeros or she did not read it correctly.*

EXAMPLE 3 "Class discussions"
(Part of core category "the teacher's role")

I assumed the role of manager of the discussion and sometimes participated, asking for further explanations or clarifying points. The episode to be presented here represents a moment in this process. It is coded according to aspects of the teacher's role in encouraging class discussion.

Teacher's behaviour	Teaching episode
Encouraging participation	I : Yiannis can you come to the board to execute the subtraction?
	Y : Okay, sir.
	Yiannis comes to the board to execute the subtraction (on decimal numbers). The student executes it as follows:
	179.983,50 - 95.265,00 ---------------- 084.718,50
Reading solution	I : [I read] : 84.718,50. What is ...
	M: [She interrupts] : Sir, I don' t agree, he should not have put a zero in front of 84.
Encouraging/Questioning	I : How does anyone else feel?
	Y : It does not matter.
Seeking clarification	I : Can you explain to us why "it does not matter?"
	Y : The number does not change.
	M: No sir, our teacher has told us not to put a zero there.
Asking for alternatives	I : What can we put instead?
	F : An equal sign.
	L : Maybe he did not know it.
Setting up contrast	I : Let's look at the two numbers the one with zero in front and the one with an equal sign in front. [I wrote them on the board.]
	G: It's the same number. The zero or the equal sign does not alter the number.
Encouraging participation	I : Can you read them Matina?
	M: Eighty four thousand ...

	[Matina read both numbers ignoring the zero and the equal sign.]
Questioning	I : Do you see any difference?
	M: No. The number remains the same.
Checking	I : Does anyone else see any difference?
	S : [all] : No.
Checking	I : So does the number change if we put a zero or an equal sign in front of 84.178,50?
	S : [all] : No.
Explaining	I : You can either put a zero or an equal sign but it does not make any difference as long as there is not a decimal point after it.

The above teaching episode had also been classified in the category "students developing arguments to support their solution methods" (part of core category "students' way of working") as well as in the category "class discussions". However, after the analysis it was selected to explain the theme of the category "students developing arguments to support their solution methods", while for the "class discussions" category a different episode was chosen.

CHAPTER 3 TEACHING EXPERIMENT-1 (TE-1)

3.1 Preliminary observations

3.1.1 The importance and usefulness of the preliminary observations

My research explored changes in the students' attitudes and behaviour while moving from traditional class teaching to group work. This was in the context of word problem solving in two fourth-grade classes in a Greek primary school in England. Examining different possible ways of starting the field research I thought that it may be helpful at the beginning to observe the teacher taking the class with which I was going to work. Before I started to explore, introduce and experiment with students, I wanted to have an idea of what was actually taking place in the classroom during some maths sessions on word problems.

These observations appeared finally important and helpful. Reasons that justify the significance of these preliminary observations are:

First, after observing the teacher for 12 maths sessions (after arrangements with her), I formed a first impression of the approach she usually employs in teaching word problem solving. I was able to compare her teaching approach with the one I was going to initiate with students for my research. I looked at what the grade-four teacher was actually doing which was different from what I was planning. All these preliminary observations helped me acquire a framework for the class with which I was going to work. This framework which consisted of the teacher's actions and approaches, students' activities and behaviours as well as of the whole atmosphere in this learning environment was very useful in considering and exploring my approach.

Secondly, these preliminary observations allowed me to familiarise myself with the environment of the grade-four class. In the beginning I was somewhat apprehensive in the class but as I continued with my observations I felt comfortable and confident. I talked easily to the teacher and students. I had more confidence to try things out for myself. During the course I felt as though became a member of the grade-four class. Teacher and students had become accustomed to my presence and they were not bothered by it, as I shall explain below. In the following sections I describe how the fourth-grade class operated.

During the school year 1994-95, I worked with this primary grade-four class where the children's age was from 9 to 10 years old. The teacher was 35 years old with ten years teaching experience. Grade-four used one of the classrooms on the ground floor. In this grade there were a total of 14 children, 9 boys and 5 girls. I observed this class for three consecutive Mondays during December 94, before the Christmas holidays, and also for nine sessions (on different days) during January - March 95.

3.1.2 The key word strategy

My first two visits to the grade-four class, the students had been given 3 or 4 straightforward problems (simple translation problems on subtraction) for homework. The students had prepared the assigned problems and they had to solve the problems on the board. This classroom setting changed gradually becoming more interesting for me, as the students started to solve problems in the class.

In 10 of the 12 sessions I noticed that the procedure in solving the word problems was the same: The teacher asked for a volunteer to come up to the board. Another student read the problem while the one at the board (after having written the data, i.e. givens, unknowns) responded with the correct operation. When the students appeared to be having difficulty in solving the problem, the teacher provided them with key words (e.g. 'altogether' suggested the students use addition to solve a particular problem) as they worked through the various problems. I often observed this happening and I was interested in finding out more from the teacher. One day after a maths session I questioned her, concerning the use of 'key words'. The following is an extract of this interview.

I : I noticed that when a student comes to the board to solve a problem and gets 'stuck' you try to help him/her by (re)reading or stating the problem again emphasising some key words.
T : Yes, with a different tone in my voice.
I : Do you do this often?
T : Yes, yes.
I : What's the purpose of doing this?
T : To help the student recognise what operation to use.
I : Do you think that helps the students?
T : Any help from the teacher assists!

My interpretation of the teacher's method is that she considered the key words as a hint or as an indirect indication to the students for the appropriate operation. 'This key-word strategy unfortunately is occasionally taught by well meaning teachers who are not aware of its limits' (Sowder, 1989, p. 151).

I think that the key words method prevents students from developing a broad conceptual understanding of the operations. Students following the key word rule may run into trouble as the indicated operation is not always the appropriate one (see e.g. Nesher and Teubal, 1975), or they may experience difficulties in finding a solution when they cannot identify key words in the problem.

3.1.3 The absence of 'why' questions

The title of the unit appeared to be a determining factor in choosing an operation. Students knew (from the title) what the unit was about (e.g. multiplication, division etc.) and often used the operation indicated without reading and visualising the problem. Although many students used the unit's title to decide on operations instantly, without making an effort to understand their choices, their actions were perfectly consistent with the goal of being able to solve the problem.

These observations were justified from the answers of a student (Petros) who was sitting next to me. When I asked him if he would like to answer a couple of questions about the day's lesson, he agreed willingly. At the end of the session, while the rest of the class went for recess we stayed in our seats and the following short interview took place:

I : I noticed that you also solved the second problem correctly.

P: Yes.

I : How did you conclude that you had to use a subtraction?

P: All these [he indicates to me four problems in the book] are subtraction problems.

I : How do you know it?

P: Today we are doing subtraction. [The unit's title was: 'The operation of subtraction - We ask for the difference' (Ministry of National Education and Religion, 1993a, p. 78).]

I : All right. I agree with you. But in the problem I am asking you why did you use a subtraction?

P: [no response]

I : Can you explain me why you used a subtraction and not another operation?

P: I know that I have to do a subtraction.
This is the correct operation for this problem.

I : Yes, but why?

P: We don't ask our teacher why. We have only to point out the correct operation.

From the last two answers given I had the impression that from student's perspective it was not necessary to know 'why' he would apply that particular operation (Skemp, 1989). I think that it is important for students to be able to provide justifications for the operations they choose. This can demonstrate if students have understood the operations and if they have grasped the problem.

The students' typical behaviour in problem solving, that is, indicating operations without explaining why, and only showing an interest in the correct answer seemed to be consistent with their teacher's way of teaching. The pupils appeared to satisfy their teacher's expectations following the didactical contract (Brousseau, 1984) that had been established.

The teacher asked the following common questions: (i) "What operation are we going to use?" and (ii) "What result did you get?" She addressed the first question when the students started working on a word problem and the second after the students had solved it. Almost all the lessons I observed started with either of these two questions. Their frequent use intrigued me and I wanted to find more from the teacher. One day, after a lesson in which she had used them most I approached her and we had a discussion about these two questions. The following is an extract from this interview with the teacher, that took place at her desk, in the classroom.

I : I noticed that you often ask students: "What operation are we going to use?" and "What result did you get?" What is the purpose of these two questions?

T: I want to check up how they worked and what answer they found.

I : Do you often ask these questions ?

T: Yes, most of the time.

I : Is the indication of the operation important for you?

T: Of course, but more important is the correct answer. This is what counts most.

I : Why is the correct answer important?

T: You can see if the students understood and solved the problem.

I : Do you ask students why they chose a particular operation?

T: Sometimes.

I : Don't you think that this is more helpful for assessing their understanding?

T: I think that my way works fine.

This particular teacher, at least in the sessions I observed, mostly used the above two questions. I noticed that she did not use 'why' questions. As suggested by Schoenfeld (1985), I believe that students should be encouraged to explain and justify their choices of operations. Regarding the teacher's approach it appeared to me that there was more emphasis placed on the students providing the correct answer rather than on developing an understanding. As was indicated by the interview the correct operations and mainly the correct answers were what counted most for the teacher. I think that the correct answers are not enough to demonstrate students' mathematical understanding as they do not necessarily imply that they have understood the problem. Conversely, neither do incorrect answers necessarily imply that students do not understand the problem.

The teacher's role appeared very rigid during my observation block. Mathematical discourse in the classroom was procedural. Discussion was restricted only to the kind of operations to be used in the problem as well as in the result/s. When the solution to a problem was written on the board the teacher's instructive role usually stopped there, which was a characteristic during these twelve preliminary observations. The students were not presented with questions or activities that would stimulate their level of understanding of the problem and its solution. Different solutions from the solved problems were very rarely explored. When most students had difficulties with a word problem the teacher solved it by 'exposition' (Burkhardt 1988, p. 17) on the board. The same method, 'exposition', was employed when a problem was used for teaching something new. The teacher did not use the problem as a challenging task, for making her students think.

Most of the time the teacher started the maths lesson by asking the students: "Who wants to read?" or by telling the class "We will continue with the next unit". At the end of the lesson her usual comment was "For the next lesson we will go over these two (or three) pages." This classroom teacher tended to go through the maths unit as it was explained in the textbook. The students seemed to read the book like a story. It is my opinion that she did not make sufficient effort to stimulate the students' interest. She would assign two or three pages to students regardless of how much work these required, or what progress the class had made that day. The teacher left me with the impression that she was following the 'transmission teaching style' and that her only concern was covering the mathematical syllabus rather than stimulating students' understanding.

3.1.4 The students' typical behaviour

A common observation in this fourth-grade class was that although the students who were coming to the board to solve a problem could perform the operation/s correctly, they were not able to explain their answers. They could not justify their answer or what it stood for or really meant. Therefore, the answer to the problem was given by another student or by the teacher. An example of this follows. Georgia, a grade-four student, came to the board to solve a problem.

T: Georgia what result did you find?

G: 13.575

T: Can you say what is the 13.575?

G: [no response]

T: Can you explain what 13.575 stands for?

G: [again, no response]

T: Who wants to give the answer to the problem?

E: The tour cost 13.575 drachmas.

In addition, when students came to the board they did not take any responsibility for solving the problem and were more likely to work under the close guidance of the teacher. In thirty two problems which I saw, the students were not encouraged to work through and develop their own solutions. Instead they were influenced by the teacher's method of reaching the solution.

Furthermore, when a student solved a problem on the board (with the teacher's attention concentrated on it) only a few of the rest of the class were watching. I felt that what counted for students was who was going to come to the board. If they were not chosen then they preferred to engage in unrelated activities until the next problem arose. When I asked for the teacher's explanation to this students' behaviour the reply was:

I : I have noticed that when you ask for a volunteer to come on the board, most of the students want to come. How would you explain that?

T: They like to use the board and the colour markers.

I : Why?

T: Because they are not allowed free access.

I : What do you mean?

T: I don't allow my students to use the board and the markers during the break because not only they mess everything but also I may loose the markers.

I : Did this ever happen before?

T: Yes.

...

I : Is this the only reason they want to use the board?

T: They want to 'feel' also that they are the teacher. When they come to the board I encourage them by saying 'you are the teacher now.'

I : What is the purpose of this encouragement?

T: When pretending that they are the teacher, students show the rest of the class how to solve the problem.

I : Anything else ?

T: Also they like to be rewarded.

Carefully observing the students' behaviour during problem solving, it appeared that for these grade-four students, this meant selecting an appropriate operation leading to the correct answer rather than trying to understand the situation first and then analyse it. Their emphasis was more on key words or rules for specifying the necessary operation. They tended to use operations as reasons for actions without recognising that using an operation is different from explaining or justifying why it is legitimate to use it in a particular problem (Lampert, 1990). Students' actions were limited only at an instrumental level rather than at a relational level (Skemp, 1989). When the students solved a word problem in class they worked individually although it was not asked by the teacher. The competition among them, at least in maths sessions I observed, seemed very high. Cooperative work in these sessions was non-existent.

3.1.5 The researcher's presence in the class

I do not think my presence in the classroom was a problem for the children as I had been in their class the year before making some observations. I would say that they liked me to be with them and most were very willing to help me in whatever I needed. They knew me and also knew, in general terms, the reason for my presence. During the maths sessions they appeared involved in their work and in their teacher's instructions and most of the time my presence seemed to be unnoticed.

The teacher on the other hand, during my first observation, I think was a little nervous. In everything she said or did, she had to make a reference to me. The first time I visited her class I got the impression that she had informed the students beforehand of my coming. The next two times, after I had talked to her more analytically about my research and I had explained the procedure that I would follow, she seemed more relaxed and comfortable. After my third observation I did not notice anything stilted in the teacher's comments or movements. I found her behaviour natural. However, she decided not to become involved in the actual teaching stage of the teaching experiment.

3.2 Design of instruments

3.2.1 Design of the pre-test and pre-interview

Prior to the beginning of my actual teaching stage in teaching experiment-1 (TE-1), a pre-test was administered to all students (15) in the fourth-grade class and an interview was conducted with all but one of the students in the class.

Design of the pre-test

The test was on the unit 'How we solve problems of four operations' which students had not been taught. This unit was taught after the test and was the next in the grade-four textbook. The test included three word problems. The first was a routine problem, like those commonly encountered in mathematics lessons and in the textbook. The other two appeared to be routine but they were problems of a type that grade-four students encountered for first time (Appendix 3). The purpose of this test was to include problems whose solutions required more than one operation (Table 3.1). The students had no experience in solving these problems.

Table 3.1 The type and number of operations required for each problem on the pre-test

Pre-test		
Problems	**Type of operation required**	**Number of operations required**
1st problem	multiplication	1
2nd problem	multiplication addition	3
3rd problem	division	2
	subtraction	

Each problem was scored with respect to the degree of understanding, the planning that was apparent, and the correctness of the answer. The explanation and justification for the solution method provided during the interview was also taken into consideration.

Design of the pre-interview

The interview was conducted after the students had performed the written test on problem solving. The interview was about the problems in the test and all students who had solved at least one of the three problems, were asked two questions: (1) "What operation/s did you use?" and (2) "Why did you use this operation?" The purpose of these two questions was to prompt the students to explain and justify the solution method they followed. The interviews took place in the second floor of the Greek school in a very quiet room that is used for drama.

3.2.2 Design of the post-test and post-interview

Design of the post-test

A written post-test (Appendix 4), different from the pre-test in terms of the problems, was administered to all but one student within a week of the end of my actual teaching stage.

Table 3.2 The type and number of operations required for each problem on the post-test

Post-test		
Problems	**Type of operation required**	**Number of operations required**
1st problem	multiplication	2 or 3
	addition	(depending on the solution)
2nd problem	division	3
	subtraction	
3rd problem	addition	2
	subtraction	

The test included three word problems that students had never encountered before. In addition, the level of difficulty of the problems in the post-test, compared to the problems in the pre-test, was greater (Table 3.2). All the problems, as in the pre-test, were scored with respect to the degree of understanding, planning and correctness of the answer presented in the solution method as well as with regard to the explanation and justification provided during the interview.

Design of the post-interview

A week after the end of my teaching experiment I administered to students a post-test with three word problems that they were not familiar with. Again, as in the pre-interview, the post-interview procedure started immediately after the students had written the test. During this interview they were asked to explain and justify their solution method. The interviews were conducted with 14 of the 15 students. One of them had left the class for the rest of the school year with special permission. Both the pre-interviews and post-interviews were conducted in the same room.

The purpose of this interview was to study the students' approach to three word problems. The students had been taught by me ten units on word problems using my alternative teaching strategy. Therefore, my intent in the post-interview was to examine if students would treat word problems differently than before the treatment. I hoped to study this through their explanations and justifications of the word problems included in the post-test.

3.3 Selection of groups

In this section I discuss the conditions under which the teaching experiment took place. Specifically I explain how I established mixed ability groups in class and I describe how I outlined my teaching approach to the students.

3.3.1 Establishing mixed ability groups and classroom organisation

The children in this study were organised into mixed ability groups on the basis of:

(i) my experience from the preliminary observations,
(ii) the pre-test that the students did individually and
(iii) the pre-interview conducted on an individual basis.

(i) Preliminary observations: The preliminary observations provided me with an opportunity to observe the students and teacher informally within their classroom setting. I was able to note the behaviour and attitudes of the individual children, how they worked and interacted and also with their classroom teacher.

(ii) Pre-test: The children were asked to solve three word problems during the pre-test with which they were not familiar.

(iii) Pre-interview: The interview was about the problems on the test and the students were asked to explain and justify their solution methods. The pre-interview which followed the pre-test provided me with an even better overall impression of the students' understanding of the word problems.

As a result of all the above criteria the children were placed into three categories: high - medium - low. I believe this was a fairly accurate method to determine which ability category particular students met. Then I formed the groups deciding that each one would consist of children from the high category, the medium category and the low category (Figure 3.1). That is, I selected one student from each category (high - medium - low) at random and ensured that all four groups were of mixed sex.

Figure 3.1 The mixed ability groups

Group 1		Group 2		Group 3		Group 4	
	Ability level		Ability level		Ability level		Ability level
Yiannis	H	Litsa	H	Elias	H	Anna	H
Georgia	M	Stavros	M	Matina	M	Fotis	M
Koula	M	Petros	L	Tasos	M	Nikitas	L
Vasilis	L	Dina	L	Costas	L		

Originally, I had thought to examine mixed ability groups with a narrower range of ability, like medium-ability and high-ability students or medium-ability and low-ability students. There is research evidence that 'all students in these groups tended to be active participants, with questions eliciting help more frequently than in mixed-ability groups with a wider range of ability' (Webb, 1991, p. 379). However, the limitations imposed on me from the maths textbook and the parents' expectations were determining factors in deciding about the composition of the group's ability. Prior to the teaching experiment-1 I had discussed with the regular teacher and we had agreed that my research was to be conducted using the word problems which were included in the grade-four maths textbook. Furthermore, all the students in class would work on the same problems.

Quite early, it was made clear to me that teacher and parents regarded the covering of the mathematics syllabus as one of their highest priorities. Assigning to students word problems not included in the set textbook or even giving less work to low-ability students was not acceptable to the parents as they made it clear from their interventions to the teacher. In addition, classifying the students in mixed ability groups with a narrow range (i.e. medium-ability and low-ability students) or in homogeneous groups (i.e. low-ability students) could raise unexpected problems. For example, allotting students in high-ability groups (high-ability and medium-ability students or homogeneous of high-ability) would be welcomed by the parents but placing their children in a low-ability group was perceived as unacceptable and incompatible with their perception about schooling and progress.

I was sensitive to the school's equal opportunities policy and tried to form groups of mixed sex. I decided to put aside all preconceived ideas about who works well with whom, and insist on the groupings based on the above criteria. I had to explain to the children that they would work in some groups chosen by me so it was not envisaged that there would be strong objections. I was aware that certain individuals would not work well with others, but I wanted to give each child an equal chance to take part in their allotted groups.

3.3.2 Explaining to the students the teaching approach

From the beginning I made clear to the children what I expected of their groups and what I wanted them to do. I took time to stress to each group their autonomy, and to give each group a chance to develop a social identity, where individuals make a contribution to a joint outcome. I started with problems where the solution required one operation and then broadened the scope to those requiring more operations. During this time, I tried to make my teacher talk, or feedback, critical about the quality of the work, rather than evaluative which only corrected mistakes. I tried to make whatever evaluative comments I had about the work to the class as a whole and not mentioning individuals.

On the very first day of my actual teaching stage in teaching experiment-1 (TE-1), I brought a poster with two questions on it and I asked three students to read them out loud. The two questions were (Schoenfeld, 1985, p. 374):

'What are you doing?' (What operation are you using?)
'Why are you doing it?' (Why are you using this operation?)

After I initiated a discussion about the questions and I explained to students that when they had worked on a problem I would expect them to explain the type of operation they used and provide a reason for it. I encouraged them to collaborate with their group-mates, try to explain to them their thinking, and help each other during the problem solving process by asking and answering questions (providing feedback and support). Webb (1991) discussing the research's implications for influencing interaction in the group, suggests that 'a more direct method is to give students direct instruction or training in giving explanations to each other and in trying to respond appropriately to calls for help' (p. 383).

I continued solving a word problem on the board. I demonstrated my thought process with respect to the method paying particular attention to the answers of the above two questions. During this time the students behaviour suggests that they were concentrating on the explanation while actively engaging in their own thought process. This was more evident with the high-attainers in the class, as after I finished explaining my reasoning they offered alternative explanations. I then divided the class into four small groups of mixed sex and ability. Three of these had four members in each and the fourth had three members.

During a typical lesson I asked the students to work in their groups to solve the problem I assigned them. Then I held a whole class discussion giving each group the opportunity to explain their solutions to the problem they had encountered. I allowed groups an open forum in which they could explain their method of solution and compare this with those of the other groups.

My preliminary observations provided me with some feedback as to the children's typical behaviour in word problem solving, which basically was to identify key words for specifying the proper operation/s. Consequently I was expecting some reaction from them to this new approach. At the same time I was interested to explore with students the effectiveness of this approach to teaching and learning word problem solving.

3.4 Analysis of the actual teaching stage

In this section I present and discuss the findings of the actual teaching stage in TE-1. These findings are grouped under three themes:

- students' way of working (section 3.4.1)
- students' attitudes to word problems (section 3.4.2)
- the teacher's role (section 3.4.3).

Sequences of classroom dialogue, together with contextual information, concerning what the teacher and pupils were doing while actively working, are also presented to explain these findings. All the excerpts provided here are actual transcripts. The names of the children have been altered to protect their identity. In the dialogues the children are identified by the initial of their altered names (Figure 3.2). The teacher (researcher) is identified by the single letter 'I'. My aim has been to present these sequences of talk as accurately as possible, using some conventions for the transcription of discourse, but at the same time ensuring that they remain easily readable and comprehensible.

Transcription conventions

________ Intonation / Emphasis

[ ] Contextual information / Explanation

Figure 3.2 Identification letters for groups and students

Group1	G1	Group2	G2	Group3	G3	Group4	G4
Yiannis	Y	Litsa	L	Elias	E	Anna	A
Georgia	G	Stavros	S	Matina	M	Fotis	F
Koula	K	Petros	P	Tasos	T	Nikitas	N
Vasilis	V	Dina	D	Costas	C		
Students as a group : **S**							

3.4.1 Students' way of working

Before students engaged in group work I explained that I wanted them to collaborate with their other group members and try to explain all their thinking to their fellows. They were asked to indicate the type of operation they were going to use and provide a justification for their choice. I emphasised to the students the importance of all group members being able to explain and justify the solution method and so I discouraged them from dividing up the problems into pieces. In this section I will discuss the students' collaboration, revision of their thinking, incorporation of others' solutions, development of arguments to justify their solution methods and finally the establishment of classroom norms.

3.4.1.1 Students demonstrating their collaboration

The data collected suggests that during class discussion, when I asked a group to tell the class how they thought about the problem, all members made an effort to offer the best explanation and justification. An example of such a behaviour comes from the next problem: 'A bus transported in a week 438 passengers and collected 98.500 drachmas. What is the cost of each ticket?' (Ministry of National Education and Religion, 1993a, p. 112). The following extract, from the class discussion about this problem's solution, illustrates the collaborative efforts of the group members:

I : Let start with someone from the first group (G1).
K : We will use a division.
I : What are you going to divide?
K : The 985.000 drachmas by 438.
I : Can you explain to us why you will use a division?
K : [She did not respond as another student, Georgia, from the same group intervened with the answer.]
G: In order to find how much each ticket costs.
K : We want to find the price of the ticket that each passenger paid for.

[When Koula finished her explanation, Yiannis concluded] :

Y: When we are given the 'many' and we want to find the 'one' we are doing division.

I : [to the class] : Do you all agree with what this group said?

S : [all] : Yes.

[Then Elias, a high-attainer from the third group (G3) commented] :

E : Sir, they explained and justified their solution method not perfectly but very perfectly.

[After, Petros, a low-attainer from the second group (G2) explained] :

P: Sir I agree. I understood the problem.

In the above episode Koula did not manage to answer 'why' she used a division because Georgia from the same group, intervened providing the justification. However, Koula did not give up her effort but offered her explanation after Georgia had finished. Koula did not seem to be bothered by Georgia's intervention as she probably recognised that all members' efforts are counted collectively. After Koula and Georgia finished their explanations, Yiannis stated the rule for using division. I noticed in the episode that the members of this group justified their solution method, both in terms of the situation involved in the problem and of a mathematical rule.

The arguments provided by Koula, Georgia and Yiannis (G1), to support their solution indicate that all group members had developed an understanding of the situation. During the presentation, the students appeared to intensify their efforts in order to sufficiently explain and justify their solution. The students were involved in such an activity for the first time and they may have wanted to give a good picture to the rest of the class. However, this behaviour was demonstrated by all groups throughout the actual teaching but not necessarily by all their members.

Furthermore, the above extract indicates that when a group took the lead in explanation then the other groups listened without interruption or intervention.

Consistent with this behaviour Elias and Petros, from G3 and G2 respectively, made their comments after the group members of G1 had finished with the presentation of the solution method.

3.4.1.2 Students revising their thinking

When I asked students to explain their solution method and they discovered errors, they quickly revised their thinking. The following is an example of such a case after the students had worked on the following problem: 'A bookstore sold 125 mathematics books for 2.500 drachmas each. How many drachmas did the bookstore take?' (MNER, 1993a, p. 53).

I : Stavros, what did you use in this problem?

S : Division.

I : Can you explain to us why?

S : The problem gives us the price of 'many' and asks us for the price of 'one'.
[I introduced this relation of 'one' and 'many' in an earlier lesson. Although Stavros provided an inappropriate justification I did not intervene.]

I : What did you divide?

S : The 2.500 by 125.

I : Would you like to show us on the board?
[Stavros came up to the board and performed the long division. Before he started executing the operation, he concentrated and commented] :

S : No, it's wrong. We must use multiplication.

I : Why must we use multiplication?

S : Because the problem asks for the cost of all books.

I : Can you tell us when we use multiplication?

S : When we know the price of 'one' and we want to find the price of 'many'.

I : Can you relate that to the problem?

S : We know that the price of one book is 2.500 drachmas and we want to find the price of 125 books.

Stavros came to the board and wrote the calculation (2.500/125) but before he started to execute it, he changed his mind and responded with a strong reaction: "No, it's wrong. We must use multiplication." When I asked him about the use of multiplication, he responded in a way that indicates the student had grasped the situation described in the problem. The response "Because the problem asks for the cost of all books." suggests that this particular student had viewed the "... cost of all books" to be equivalent to the problem's question, i.e. "How many drachmas did the bookstore take?". The student constructed his own interpretation of the problem and he answered my question expressing the situation from his own perspective.

This episode suggests that reconsidering a solution method and finding it inappropriate, helped the student to revise his thinking towards the correct method. When students provided inappropriate explanations or inappropriate justifications I did not interrupt nor discourage them from proceeding further. On the contrary, I encouraged them to complete their thinking or their calculations on the board. This appeared to be an effective strategy as it helped students to identify their own mistakes, revise their thinking and finally work through to the correct answer. In addition, I should point out that students demonstrated similar behaviour during the pre-test and post-test when asked to explain their solution methods.

3.4.1.3 Students incorporating other students' solutions

One of the dominant categories which emerged from the data was relevant to the likely effects upon children's understanding of having to attend to the solution method of another pupil from a different group.

The students heard their classmates justifying their solutions and tended to revise their thinking in light of a correct method. The following is an example from a group of students who revised their incorrect method as they listened to the explanation of a student from another group. The students have been assigned to work on the following problem: 'A theatre of mime gave two performances in a school. The first performance was attended by 180 children and the second by 240 children. The price of the ticket was 500 drachmas. How much did the theatre make?' (MNER, 1993a, p. 54). After the students had worked on the problem, they made the following comments about the solution:

F : We will do a multiplication, 180 by 500 and after we'll multiply what we find with the 240. And then we'll add these two.
[Fotis said what he would do but he did not explain why. While he was describing his solution method, I coded the specified operations on the board.]

S : Sir we disagree. We'll multiply 180 times 500 because 180 were the spectators and 500 drachmas were the price of the ticket. Doing this we'll find how many drachmas the theatre collected from the first performance. After, we'll do another multiplication 240 times 500 to find how many drachmas were raised after the second performance. Then if we add the money from the first performance and the money from the second performance we'll get how many drachmas were made in total.
[Again here, while Stavros explained and justified his solution method I coded the specific operations on the board.]

F : Sir, can I say something?

I : Yes, Fotis.

F : There, [pointing to the board] we shall multiply 240 times 500.

I : Before you said that you were going to multiply the result from the first multiplication times 240.

A : We made a mistake.

I : Can you explain to us why it's a mistake?

F : Because it does not make sense. You will not get anything if you multiply the collections from the first performance by the price of the ticket.

A : Sir we did not think carefully. We were in a hurry.

I : Can you tell us then why you are going to multiply 240 times 500?

A : Because the second performance was attended by 240 children and the price of the ticket was 500 drachmas. We want to know how many drachmas the 240 children paid in all.

In this extract I noticed that although Fotis (G4) originally, specified the mathematical operations for the problem he did not justify them, while Stavros (G2) presented and justified his group's solution appropriately. After Stavros had finished with the explanation, Fotis and his group-mate Anna commented on a change in the solution method they presented earlier. They were clearly aware of their mistake and provided a reason for it.

When Stavros presented the solution method of his group, it did not take long for the students of G4 (Anna and Fotis) to reconsider their solution method. This suggests that the students having worked together on the problem, and being involved in the situation were able to consider and incorporate the solutions of their classmates. Working in groups and explaining their solution method was a new experience for these students, as with their regular teacher were not justifying their solutions, rather they seemed to choose operations and perform calculations. However, class discussions seemed vital in enabling the group-mates to revise their solution method and to incorporate others' methods. It seemed important for the children to reflect upon how successful their way of approaching the solution had been and how they might differently approach solutions next time.

At other times, thinking about other groups' solution methods and utilising them in their own thinking helped group members decide on their appropriateness and correctness. The students had been given the following problem, to be completed in class: 'On the opening day, a cinema collected 469.200 drachmas and 448.800 drachmas on the second day. Its expenses (film renting, staff wages, tax etc.) were 235.00 drachmas the first day and 202.000 drachmas the second day. How many drachmas was its profit?' (MNER, 1993a, p. 118).

While the students were working on this problem, I went from group to group. I realised that two of them (G2 and G3) had approached the solution in the same way, another (G4) had followed a different path, and the last group (G1) had solved the problem using yet another strategy. When I asked students to demonstrate their solution methods the following discussion took place:

I : Which group would like to start?

E : Sir, can we start?

I : Yes, Elias (G3).

E : First we subtracted the 235.000 drachmas, the expenses for the first day, from the 469.200 drachmas which were the collections for the first day. So we found its profit for the first day. Then we subtracted the 202.000 drachmas from the 448.800 drachmas to find the profit for the second day. After we added the profit from the first day and the profit from the second day we found its total profit for both days.

After the explanation, Matina and Tasos, from the same group came to the board and executed the operations correctly. They found the total profit to be 481.000 drachmas. Then Litsa, from a different group (G2), which had solved the problem in the same manner, insisted in taking the lead in the explanation.

L : Sir, we followed exactly the same route. First we subtracted the 235.000 drachmas from the 469.200 drachmas and after we subtracted the 202.000 drachmas from the 448.800 drachmas. From the first subtraction we found how much money was left over from the first day and from the second subtraction we found how much money was left over from the second day.

I : What do you mean by "how much money was left"?

L : It is the net income, the profit. The amount that was left over after the expenses had been paid. Then we added the profit from the first day and the profit from the second day and we got the total profit, that was 481.000 drachmas.

When Litsa finished the explanation, Anna from another group (G4) which had solved the problem in a different way commented:

A : Sir, Elias and Litsa presented one way of solving the problem. Although we could find the profit as they did it, we thought about it and we decided to do something simpler.

Yet Yiannis from the last group (G1), who had solved the problem using two ways, intervened:

Y : Sir we agree. There are two ways of solving this problem.

...

I : Anna would you like to share with us how you approached the solution to the problem?

A : We added the income from the first day with the income from the second day. So we had the total income from both days. Then we did the same with the expenses. By adding the expenses for the first and the second day we found the total expenses. When we had the total income and the total expenses we subtracted the expenses from the income and we found the total profit for the cinema.

[Then Georgia from G1 commented] :

G : Sir we solved the problem with both ways and we got the same profit.

Two students from G1 (Koula and Yiannis) came to the board and performed the calculations for the required operations of the second method. Then Elias from G3, who had solved the problem finding the profit for each day separately, took the lead.

E : We agree that both ways of solving the problem are correct. If we had added the income for both days and the expenses for both days then we could find the profit ... [pause] the total profit ... [pause] subtracting the expenses from the income.

As is apparent in the above extract, two groups (G2 and G3) solved the problem following the same solution method, another group followed a different one (G4) and the last group (G1) solved the problem in two ways. In the beginning Elias and Litsa presented and justified their respective group's solution method in an appropriate way. When G4 took the lead with Anna, I noticed that she first credited Elias and Litsa for their solution method and then she commented that her group followed an alternative solution, providing also an explanation: "We decided to do something simpler." This implies that Anna's group, while being aware of the calculations involved in the solution just presented, decided to follow an alternative method with simpler calculations.

It is characteristic that although Anna had not yet presented her group's method and therefore it was not known to Yiannis (a high-attainer), he nonetheless intervened and agreed that there are two ways to solve the problem. Yiannis, being aware of Elias and Litsa's solution and comparing it with their own, asserts that there are two ways of solving the problem.

The above extract demonstrates the students' competence in incorporating their classmates explanations and justifications. As a result they could also decide on the appropriateness and the correctness.

It could be argued that during the actual teaching stage, when the students were engaged in the problem, knowing the given of the problem, what was asked, as well as the relationships involved in the situation, it seemed easier for them to incorporate the other students' solutions. They did not appear surprised by the different solutions. It did not take long for Elias to agree that both solution methods which demonstrated in class were acceptable.

One of the strengths of class discussion was that it necessarily faced the group-mates with solution methods different from their own. In my view, if the members of a group are to achieve anything more than a solution method it is necessary to take such other solution methods into account and from them to build up a more complex model. This seemed to be the strategy used by the groups. Instead of rejecting another group's solution method as 'irrelevant' or 'wrong', they used it with modifications, to become part of a shared understanding.

3.4.1.4 Students developing arguments to support their solution methods

Providing arguments to justify a solution method proved to be a strategy that was followed and established by all groups. The students had to provide evidence to support or reject an assertion rather than saying simply that an answer is right or wrong. My 'why' question may have influenced them towards this behaviour. When someone commented that a solution method should be eliminated because it was incorrect, I asked this student to explain 'why'.

When a pupil was asked to explain the solution method of his/her group in class, he/she tended to start with a discussion during which all the members of the group tried to contribute, in order to provide an appropriate justification. The next problem is an example that demonstrates this behaviour.

The students were given the following problem to work on: 'The pedestrian walks at a speed of 4 km per hour. The truck's speed is 14 times the speed of the pedestrian. The speed of the aeroplane is 8 times the speed of the truck. What is the aeroplane's speed?' (MNER, 1993a, p. 57). During the discussion about the solution, the following comments were made:

I : What did you do?
E : Multiplication, 14 times 4.
I : Could you explain that a little further?
E : In order to find the truck's speed.
M: Since we want to find how many km is the truck's speed.
I : We agree. We'll use a multiplication since the truck runs 14 times faster than the pedestrian.

I : ...

T : [Tasos came to the board and performed the multiplication, 14x4=56.]

I : What did you do after you found the speed of the truck?

C : We multiplied 14 by 8 and we found that the plane's speed is 112.

G : Sir we don't agree. The problem says that 'the aeroplane's speed is 8 times the speed of the truck'. So we must multiply 56 by 8.

Y : Yes sir. The truck's speed is not 14 but 14 times 4 which we found to be 56.

I : Can you put all these in one answer using the units when you refer to speed?

K : Sir, since the plane's speed is 8 times the speed of the truck, we multiplied 8 by 56 km, which is the speed of the truck per hour, and we found that the plane's speed is 448 km per hour.

Elias, Matina, Tasos and Costas multiplied 14 times 4 to find the speed of the truck and they explained and justified the operation in an appropriate way. The students seemed to be aware of the problem's statement that 'The truck's speed is 14 times the speed of the pedestrian'. Tasos came to the board and performed the calculation finding that the truck's speed is 56 km per hour. When I asked this group to say what they did after, Costas responded by saying that they multiplied 14 times 8 and they found the plane's speed to be 112. Costas' response implied that the students of this group had been misled by the previous statement, that is, instead of reading the problem's statement as it is i.e. 'The speed of the aeroplane is 8 times the speed of the truck.', they had read it as 'The speed of the aeroplane is 8 times the speed of 14', implying that the truck's speed is 14. Although this group offered an inappropriate explanation I did not make any evaluative comments but waited instead for other groups' responses and reactions.

Immediately, Georgia, Yiannis and Koula examined their classmates' assumptions and they disagreed, responding in a polite manner. They participated in the conversation, refuting the first's group (G3) assertion, providing an explanation for that. Georgia did not start her explanation of their solution method from the beginning, but she focused at the disagreement point. She read the statement of the problem correctly and then specified the operation.

The above extract is also an example that shows students' collaborating. The way that the two groups (G3 and G1) presented and justified their solution methods indicates that the collaboration of the group members was not restricted only to the group work but was extended to classroom presentations.

At other times when the students met points of conflict in the process of problem solving, they resolved them through class discussion where the individuals provided arguments. The episode to be presented here represents a moment in this process. The students had been assigned to solve a word problem on decimal numbers. The problem included additions and subtractions. Although the students did not have any difficulty with the additions (the numbers had been placed in vertical order in the book) they found a point of dispute at the subtraction.

I : Yiannis can you come to the board to execute the subtraction?
Y : Okay, sir.

Yiannis comes to the board to execute the subtraction (on decimal numbers). The student executes it as follows:

```
  179.983,50
-  95.265,00
------------
  084.718,50
```

I : [I read] : 84.718,50. What is ...
M: [She interrupts] : Sir, I don' t agree, he should not have put a zero in front of 84.

I : How does anyone else feel?

Y : It does not matter.

I : Can you explain to us why "it does not matter?"

Y : The number does not change.

M: No sir, our teacher has told us not to put a zero there.

I : What can we put instead?

F : An equal sign.

L : Maybe he did not know it.

I : Let's look at the two numbers the one with zero in front and the one with an equal sign in front.

[I wrote them on the board.]

G: It's the same number. The zero or the equal sign does not alter the number.

I : Can you read them Matina?

M: Eighty four thousand ...

[Matina read both numbers ignoring the zero and the equal sign.]

I : Do you see any difference?

M: No. The number remains the same.

I : Does anyone else see any difference?

S : [all] : No.

I : So does the number change if we put a zero or an equal sign in front of 84.178,50?

S : [all] : No.

I : You can either put a zero or an equal sign but it does not make any difference as long as there is not a decimal point after it.

In the above extract the students encountered a point of dispute in the subtraction of decimal numbers. We examined this point in a whole class discussion where the students engaged in debate. In this argument the students participated not as a group but as individuals. They expressed their opinions and they tried to justify them. I assumed the role of manager of the discussion and sometimes participated, asking for further explanations or clarifying points.

Perret-Clermont (1980) outlined how peer interaction enhances the development of logical reasoning through the process of active cognitive reorganisation, induced by cognitive conflict. She claims that this is most likely to occur in situations where children with moderately discrepant perspectives are asked to reach a consensus. There is evidence in my research as I outlined earlier which gives examples of such conflict and how the individuals involved go about reaching agreement. Forcing individuals to recognise and coordinate conflicting perspectives in a problem, helps the cognitive process: 'for the task to have educational value, it is not sufficient for it merely to engage children in joint activity, there must also be confrontations between different points of view' (Perret-Clermont, 1980, p. 196). I felt that this kind of conflict was essential to the development of solution methods, not just in individual group collaboration but also to face this conflict head on in classroom discussions.

The above extract also suggests that although the students were involved in class discussion rather than in group discussion, their attitude and behaviour towards their classmates had changed. During the argument the students responded to their classmates' assertions following the class' norms (M : Sir, I don't agree, ..., L : Maybe he didn't know it). On the contrary, during the preliminary observations, the students appeared to respond to their classmates with negative comments when they had made a mistake.

3.4.1.5 Establishing norms

At the beginning of the actual teaching in TE-1, I explained to the students that they were not to use words like 'it's wrong', 'it's a mistake' (and mainly critical comments) when they compared their work with their classmates. I encouraged them to respond with phrases like 'I do not agree for ... this and this reason.' The transcription of tapes and the observations during class discussions showed that the students from the beginning of teaching experiment-1, appeared consistent with the behaviour I asked them to follow. The following extract demonstrates the students' behaviour:

I : What do you think you will use here?

G: Subtraction.

I : Can you explain to us why?

G: In order to find his profit.

I : What will you subtract?

G: The income from the expenses, the expenses from the income. 350.000 drachmas are the expenses and 60.000 drachmas are the income. I will subtract 350.000 from 60.000.

[The actual figures were 350.000 for the expenses and 600.000 for the income.]

L : Sir?

I : Yes, Litsa.

L : I do not exactly agree with Georgia. Not with the operation but how she said it. She said 350.000 from 60.000. Therefore either she forgot to put the zeros or she did not read it correctly.

Litsa responded with a moderate way against Georgia's assertion that would subtract 350.000 from 60.000. Litsa not only disagreed but justified it, providing two explanations for the logical contradiction in Georgia's assumption. Litsa did not tell Georgia that she was wrong. Instead, she left that judgement to Georgia. The challenge in this argument took the form of a logical disagreement to a hypothesis rather than a judgement of a person. The data collected indicates that similar behaviour was demonstrated by all groups during my actual teaching stage.

3.4.2 Students' attitudes to word problems

The students' attitudes to word problems and their effect on learning problem solving is important for teachers. These attitudes can influence the effort a student will put into problem solving, as well as the classroom environment and the students' motivation. In this section I shall discuss the students' approaches to word problems, their interpretation of the word problems, their self-confidence and their comments on my teaching approach.

3.4.2.1 Students' approaches to word problems

At the beginning of the actual teaching stage the students started to work in groups but they behaved as if they were feeling a bit uncomfortable about this new practice. When I asked them to engage in problem solving activities, concentrating more in explaining and justifying their solution methods, most of them seemed surprised and responded reluctantly. Evidence (from different early sessions) to support this observation includes students saying:

F : I like to solve problems but not to say why.
M: Sir can we indicate the operation but not say why?
P : I find difficult to answer the 'why'.
K : Is it not enough the operation?
T : We do not say 'why' to our teacher.
V: Again 'why'?

Asking students to provide arguments to support their solution method, after they had solved a problem, was something new to them, as they were usually ending up, until I took over the class, with the problem's solution at the finding of answers. Students' responses revealed that they were accustomed to specifying the operation/s for the problems while they were unaccustomed to justifying their choices. Only four out of fifteen students (the high-attainers) were able to answer the 'why' question, while for the rest of the students in the class to justify their choice of operation seemed an unnecessary and awkward activity. The students' refusal or inability to answer my 'why' question may be explained as a contradiction of the didactical contract which had been established between them and their regular teacher.

Brousseau (1984) explains that in every teaching situation there is a contract established

> 'which determines - explicitly for a small part, but mostly implicitly - what each partner, the teacher and the learner, has to cope with, and what is in some way or another the responsibility of one partner to the other. We are interested here in the didactical contract, that is in that part of the contract which is specific for the content, i.e. the mathematical knowledge' (p. 111-112).

According to Brousseau the didactical contract consists of three elements: the teacher, the student and the mathematical knowledge. It includes the sum of the teacher's expected behaviour from the student and also the student's expected behaviour from the teacher. It expresses the students' and teacher's reciprocal obligations to and expectations from each other which are related to the mathematical knowledge. What the student should 'know how to do'? Which mistakes should he/she avoid? What does the student expect from the teacher? What does he/she expect the teacher to ask him/her? What is allowed? What is not allowed? All these rules are valid and form a stable contract.

In the case of my teaching experiment there was a breach of the didactical contract which had been established between the students and their regular teacher. She had accustomed her students to creating a problem solving atmosphere in the classroom where they were only expected to specify the necessary operations for the problem (i.e. the 'what' question). My 'why' question which asked students to justify their choices of operations, was something new to them. It was not included in their teacher's ordinary questions. Being out of the contract, it might have been the cause of their lack of response. Therefore, the breaching of the didactical contract was inevitable as the students were faced with a 'new' and 'unusual' or 'hard' question.

According to Brousseau the importance of the didactical contract is not focused on the contract itself but on the inevitable breaches, without which it would never appear. 'In fact, learning will not be based on the correct functioning of the contract, but rather on breaching it' (p. 113).

Half way through the actual teaching stage, as the students were working collaboratively (and I kept emphasising the 'why' question), I noticed that their attitudes towards the solution of a problem gradually changed. Their responses differed significantly from those offered during the initial phase of the actual teaching. Below I cited some of their responses from the different sessions:

S : We need to understand the problem in order to explain why.
G : [to another group member] : Let us think and find out why.
Y : To say why, we have to think first.
L : Why, means to justify your operation.
T : We know why but we cannot explain it.
M: Sir I know why but I cannot describe it as it exists in my mind.

Through their responses the students pointed out two aspects of the 'why' question. First they considered that in order to explain and justify a solution method they need to have thought and understood the situation described in the problem. Their responses seemed to imply that without 'getting into' the problem it is not possible to offer a substantiated mathematical argument to support the solution method. Second students realised that although they might have an explanation within their mind for the solution method, sometimes it was difficult for them to express it verbally.

My persistence on the 'why' question helped the students to realise that they were expected to be able to satisfy a 'new' condition (the 'why' question). The students, accepting this 'new' didactical situation, began to think about the question and they tried to utilise their knowledge to produce the expected answers. At this point, the conflict of the didactical contract created at the beginning of the teaching experiment had started to resolve itself.

Towards the end of the actual teaching the students and I reached a point where everyone's behaviour had changed considerably. I had changed my way of questioning students about their solutions, while they on their part were coming up most of the times with explanations and justifications in terms of their problem solving strategies. The following is one of the extracts that demonstrates the points I made about the way I questioned students and the way in which they responded. The students had been set the following problem to be completed in class: 'A trader bought 30 blouses and paid 240.000 drachmas. He sells the blouses with profit 4.500 drachmas per each. How many drachmas will he collect?' (MNER, 1993a, p. 121). When I asked students for their solution methods the following discussion ensued:

I : Which group would like to tell us how they approached the solution to the problem?

...

Y : First we did a division. We divided the 240.000 drachmas, the money he paid for the blouses, by 30 because he bought 30 blouses. From this division we got 8.000 drachmas which is the cost of each blouse.

I : [As the student explained the solution I noted the operation/s on the board.]

I : And then? [Yiannis took the lead again.]

Y : Since we know the cost for each blouse is 8.000 drachmas and the profit from each blouse which is 4.500 drachmas, we added the cost and the profit and we found the selling price for each blouse to be 12.500 drachmas.

I : [Asking another student from the same group] :
Can you tell me Georgia what the 12.500 drachmas represent?

G : It's the money that the trader will collect for each blouse.

...

I : Who would like from the group to continue with the explanation of the solution method?

Y : After we multiplied the 12.500 drachmas, which was the price of each blouse, by 30 because the trader sold 30 blouses. So we got his total collections.

When Yiannis finished the explanation another student from this group, Koula, came to the board and executed the operations which I had marked on the board. Then I asked the other groups to comment on the solution method that was just presented.

I : What do the other groups have to say?

L : [from another group] : Sir?

I : Yes, Litsa.

L : We followed the same way. We know that the trader paid 240.000 drachmas for 30 blouses. So we divided the 240.000 drachmas by 30 and we got the cost for each blouse, that is 8.000 drachmas.

I : [Asking the other group members] : Would you like someone else from the group to continue with the explanation of the solution?

S : Sir, can I continue the explanation?

I : Yes, Stavros.

S : We found that the cost of each blouse is 8.000 drachmas and it is also known from the problem that the profit from each blouse is 4.500 drachmas. Adding these two [cost and profit] we got how much the trader will sell each blouse. The total price for each blouse will be 12.500 drachmas. Then we multiplied the 12.500 by 30 because he sold 30 blouses, and we finally found that the trader will collect 375.000 drachmas.

Then Tasos from G3 commented that his group solved the problem following exactly the same way. Finally G4 started explaining their solution :

F : Sir, Yiannis and Litsa gave the same explanation but there is another way of solving this problem.

I : Would you like to share it with us?

F : We know sir that the trader bought 30 blouses and paid totally 240.000 drachmas. For all of them. Also we know that his profit from each blouse is 4.500 drachmas. So multiplying the 4.500 drachmas by 30 we found his total profit, which we got to be 135.000 drachmas.

I : Can you all follow Fotis?

...

I : Go on Fotis

F : So now we know that the total cost for the blouses is 240.000 drachmas and the total profit from the blouses is 135.000 drachmas. We added the 240.000 and the 135.000 and we got that the trader will collect a total of 375.000 drachmas from the blouses.

Then another student from this last group, Anna, came to the board, executed the operations correctly and justified the answers.

In the above extract there is nowhere a 'what' or a 'why' question. In contrary, the teacher's dominant question took the forms:

I : Which group would like to tell us how they approached the solution to the problem?

I : [Asking the other group members] : Would you like someone else from the group to continue with the explanation of the solution?

I : Would you like to share it with us?

Examining myself I realised that when I was asking students for their solution methods I did not use any of the original two questions ("What are you doing?", "Why are you doing it?"). Raising 'what' and 'why' questions proved to be important at the initial phase of the actual teaching stage but not subsequently since the students knew my expectations. In addition the original two questions had been substituted by the question "Would you like to tell us how you approached the solution to the ... problem?" or something similar.

The students discussing their solution methods were trying to explain all their steps and were also trying to justify them. They seemed to be involved in the problem as they knew what the situation was about and also tended to provide sufficient answers. Students appeared to have developed an alternative way of looking at the problems. Instead of treating the word problems as a set of words concealing specific operations, they treated them as problematic situations that needed to be understood first and then analysed.

In the last phase of the actual teaching stage it was not necessary to ask students the 'why' question, as they already knew what the expected answers should be like. A 'new' relationship had been created between the teacher / researcher and the students and therefore a 'new' didactical contract had been established. The students, accepting their obligations stemming from this 'new' contract, were trying to satisfy them without resentment.

3.4.2.2 Students' interpretation of word problems

This was the first session at the third week (of the total four) of the actual teaching stage. During the break I arranged the tables in class in groups, to avoid the confusion and the loss of time when the students come in. They came in and took their seats in their groups. Groups were asked to solve the first problem on page 109. 'The shop has the following collections and expenditures:

COLLECTIONS	EXPENDITURES
42.324,5 drachmas	21.034 drachmas
63.659 drachmas	31.890,5 drachmas
74.000 drachmas	42.340,50 drachmas

Can you calculate its net income?' (MNER, 1993a, p. 109).

As the students started to work on the problem I went from group to group observing their way of collaborating and attacking the problem in hand. After some time I asked them for their solution methods:

I : Which group would like to tell us how approached the solution?

L : Sir?

I : Okay, Litsa.

L : First of all we added in order to find the collections and then the expenditures. After we subtracted the collections from the expenditures ... [pause] no, no the expenditures from the collections in order to find the net income. Stavros found a different result. Dina, Petros and I we got the same one.

I : Which group would like to present its solution now?

F : Sir?

I : Yes, Fotis.

F : First we added the collections from the three days in order to find his total collections. Then we did the same with the expenditures. By adding the expenditures from the three days we found his total expenditures. Then we subtracted the total expenditures from the total collections and we got his net total income.

G: Can we present our solution sir?

I : Yes, Georgia.

G: We added the collections in order to find the money he [shopkeeper] collected from selling ice-cream and then we added the expenditures. After we subtracted the expenditures from the collections.

I : And what did you find?

G: The remainder.

I : What do you mean by remainder?

G: It's the net income for the shop sir.

Y: [from the same group, G1] : How much money left for the shop after paying the expenditures. It's the shop's profit.

I : Okay, you made it clear now.

Now let's hear and the last group to see how they approached the solution. Can you start Elias?

E: Sir we followed a different way. We thought about each day separately. So for the first day we subtracted our expenditures from our collections and we found our net income to be 21.290,5 drachmas. For the second day we subtracted again our expenditures from our collections and we found our net income to be 31.768,5 drachmas and for the third day we found that our net income was 31.659,5 drachmas. Then we added together the net income from each day and we found that our total net income was 84.718,5 drachmas.

T : [from the same group, G3] : Our best day was the second.

I : Why?

K : [From another group, G1] : Since it was the day that the shop had the best net income.

T : Because sir, the second day we had the most profit.

[Then I asked Costas who belong to this last group G3 to recapitulate the operations they used in their solution method.]

I : Costas can you repeat once more how many operations did you use in this problem?

C: Sir we did three subtractions and one addition. First we subtracted the first collections with the first expenditures, after we subtracted the second collections with the second expenditures, and after we subtracted the third collections with the third expenditures. After we had done these three subtractions, we added together our net income from each day and we found our net total income.

F : [from another group, G4] : This is another way of solving the problem.

S : [from another group, G2] : You can solve the problem like this but it takes more time.

E: It made more sense to us to find the net income from each day separately.

T: If we were the shopkeepers everyday we had to know our net income ...

[pause] our profit.

Y: [from another group, G1] : Your way is fine.

[Then we went on solving the problem in both ways on the board.]

In the above long extract I noticed that students worked collaboratively and managed to solve the problem using different methods for its solution. When I asked the students to discuss their solutions I realised three groups used the same method while one group developed a different one .

The two different solution methods presented by the groups may be considered as two different perspectives to the problem. The students looking at the situation from different perspectives, they developed alternative solutions. The three groups who solved the problem using the method of adding the three takings and the three expenditures and then subtracting their sums to find the net income, represented the conventional way of solving a problem. That is, they looked at the situation from outside in order to spot the appropriate arithmetic operations. The students regarded the problem as a group of words which concealed a mathematical operation needed to solve the specific problem. They saw themselves 'detached' from the situation and when they explained their solution method they used the third person (".. his total collections", "... his total expenditures", "... his net total income") referring always to the shopkeeper.

The fourth group (G3) approached and solved the problem in a completely different way. The students in this group appeared to develop some insight into the situation and put themselves the shopkeeper's position. Their solution method consisted of four steps. The mathematical explanations and justifications they provided (i.e. "So for the first day ... we found our net income to be ...", "... our net total income was ...") showed that in each step they were getting a valid answer, that was also meaningful to them. Their explanation implied that they considered themselves to be the actual practitioners of the situation. In other words, this group of students were able to put themselves in the specified situation and identify with the shopkeeper.

Another important observation from the above extract are the points made in the discussion among one of the students who solved the problem with the conventional way and the students who developed a different insight to the problem. Although the solution method developed by the three 'detached' groups was more efficient, with fewer calculations, the fourth group (G3) solved the problem in a different way. The student's response "It made more sense to us to find the net income from each day separately.", implies that the students of this group may have been aware of the first method but they preferred to solve the problem in a way more meaningful to them. The students treated the situation as if it were a real life situation and they identified themselves with it.

The solution presented by the fourth group (G3) appears to be in line with some new theoretical approaches for looking at classroom maths practice. Lave (1992) discussing the theory of situated learning suggests that the difficulty may not be one of finding connections between the school problems and the everyday life problems but 'making word problems truly problematic for children - that is, part of a practice for which the children are practitioners' (p. 88). Lave goes one by saying that 'given lively imaginations, it does not matter whether the problems conform to life experience, but it is important that they engage the imagination, that they become really problematic' (p. 88).

3.4.2.3 Students' self-confidence

Dina and Costas were two pupils who could be described as slow learners lacking motivation and concentration, at least in mathematics whenever were solving problems individually. I had 'spotted' these two students during my preliminary observations. They would rarely engage in classroom discussions and seemed to need much praise and encouragement in their work. In the groups into which I had placed them, they were easily dominated and rarely given the opportunity to express their point of view. They did work together with the other group members but progress was slow. Interestingly enough, over a period it became apparent that there was an increased involvement in their work that I had not previously observed. In the following diary excerpts, one can see how these individuals were interested, motivated and involved. Indeed, their performance levels throughout the group work appeared to improve beyond expectations.

Costas was normally a shy pupil who had very few friends and rarely contributed. He was the only child who had declined to participate in the pre-interview. In the initial period of collaborative group work, diary extracts revealed that Costas was having problems coping with the collaborative environment.

"Costas is not collaborating with his group members. He seems to be shy and prefers to work by himself. He does not participate in the group's discussion although he is asked to do so."

Later extracts reveal:
"Costas who often switches off during classroom discussions seems interested and involved. He listens to what is going on although he does not appear to offer his own solution method despite being asked."

The following week:
"Costas appeared happy to work in the group and began to open up much more. It is interesting to hear him express his own way of approaching the solution to the group and the group accepting him. Very good for Costas."

For Dina similar positive entries were made:
"Dina is working well with her group members. She is normally introverted and has very few friends in the group. She really seems to be benefiting from this arrangement ... Dina discusses with the other group members ways of

approaching the solution. She was very much part of the group and her thoughts were appreciated. Usually Dina works alone as she finds it very difficult to relate to her peers."

As I already mentioned Dina and Costas were classified in the low group of students. They were the students with the lower ability in their groups. Dina had generally friendly relationships with her group-mates while Costas was a somewhat shy lad. However both were very disciplined within their groups, and became increasingly work-orientated and in particular put much effort in, during the course of problem solving. Dina was always prepared to talk to her group members (high-ability and medium-ability students) about her personal experiences of the problem while she was eager to question them when she encountered difficulties. On the other hand Costas was unwilling to collaborate with his group members at the beginning and he needed some time to integrate.

Mixed ability groupings appeared to have done much to enhance self-confidence of the low-attaining pupils, Dina and Costas. Similar entries for other low-attaining pupils appeared to show that collaboration was beginning to enhance their self-image and self-esteem giving them more confidence in their learning:

"Petros is not concentrating on word problem solving and has to be constantly brought back onto problem solving by me ... Clearly he is using the group for his own benefit to help him."

During my preliminary observations I noticed that Petros in individual class work would often seek the teacher's help but now as this was not available he seeks help from others:

"Petros appears interested and involved. He is listening to the others in his group ... Petros was seeking help from Litsa and Stavros."

Vasilis who has more specific behavioural problems was the greatest challenge, in that I felt from the beginning I would have problems integrating him into a group. Indeed at first there were several incidents that were to confirm this as these extracts show. Early on in the project the diary read:

"Vasilis joined the group but soon he asked if he could work on his own. I tried to encourage him back into his group but it was clear they would ignore or even reject him ... He was not interested in problem solving and he was not keen to work with anyone else. Then he kept himself busy examining his personal belongings but soon became bored and started to interfere with the others."

During my preliminary observations Vasilis was noticed to demonstrate disruptive behaviour. However in the teaching experiment he appeared more disruptive because he did not have someone to talk to, or because he may have wanted to attract the attention of the other group members who were also involved in problem solving.

The next day's entry reads:

"Koula and Georgia are having problems with Vasilis. As the group work started, Vasilis' notebooks and pencils were all over the group's table. When Koula and Georgia asked him if he could put his books and pencils in order and be more polite he started to complain that they did not want him in the group ... Vasilis annoys Koula and Georgia with his behaviour. Vasilis was being too disruptive ... We negotiated that he would work within a different group from the next session."

The day after, Vasilis was placed in Anna's group. The diary read:

"Vasilis was trying his hardest to disrupt the group. He started by trying to annoy each member by telling them they approached the problem incorrectly. He had solved the problem at home but when Anna asked him to explain it he was unable. When the other group members worked on the problem Vasilis kept annoying them. Then Fotis asked him to stop and he started to complain in the class that they did not pay attention to him. Then he came to me and asked for permission to go to the washroom, happy with the reaction and the chaos that he had caused."

Even when I transferred Vasilis from group G1 and placed him in group G4 he continued to show unacceptable behaviour. He clearly could not or would not cope with working with others in the class. Eventually I asked him to come to my desk and work with me.

Pollard (1985) attributes pupils' behaviour to a manifestation of the pupils' attempt to maintain their self-image and self-esteem thereby enhancing their self-confidence. Working in these groups in this way appeared to enable Costas and Dina to build up their self-confidence and self-worth and I believe had an effect on the ways in which they performed throughout the work scheme. It was important to minimise the value I placed on success and by encouraging all solution methods I hoped to improve individual's self-esteem and develop self-confidence. I felt it was important to allow groups to decide how they would approach the solution to the problem and not favouring one solution method more than others. Thus I encouraged the children to consider solution methods they felt were appropriate. By developing a sense of ownership in their solution methods, I hoped this would improve their self-esteem. Vasilis was the only pupil in the class who did not manage to join and stay in a group by conforming to an acceptable behaviour.

Overall, working collaboratively in mixed ability groups helped two of the five low-attaining pupils, Costas and Dina to build up their self-confidence. Similar effects may also apply to other students (e.g. Petros) in the class but further research is necessary to confirm this as the change in their behaviour was not so obvious during the teaching experiment.

Kutnick (1988) argues that building up a pupil's academic self-concept is a complex process, more generally a combination derived from teachers' interventions and the pupil's performance. Bandura (1982) argues that a person's belief about his/her abilities to engage in activities necessary to allow designated performance levels (self-efficacy), is closely linked to a pupil's self-esteem and is highly likely to influence his/her choice of activities.

Vasilis' refusal to work in a group and the associated behaviour that permitted this, may have been the result of Vasilis' low self-efficacy as a group member, whereas the others clearly believe that they were capable of taking part more usefully. On a number of occasions, I noticed Vasilis observing the others involved in group work and I could see he was debating whether or not to rejoin his group. I think he was feeling neglected, although I asked him twice if he wished to rejoin his group, but he declined.

Interestingly early on, the collaborative work provided the class with opportunities to try out and develop the necessary skills and indeed it proved to be effective for the majority of pupils. It appeared that once the children were assured of success in their groups and they themselves were confident in their ability to participate, then groups worked more effectively. Vasilis' case was more extreme, yet it did appear that when I asked him to come to my desk and work together his confidence was raised and he continued to work for the rest of the research period.

3.4.2.4 Students commenting on my teaching approach

At the initial phase of my actual teaching stage I did not ask the students to comment on my teaching approach. Certainly, when I was asking the question "why ...?", their facial expressions, their responses and generally their whole behaviour in class, were revealing their attitude to this new approach. I thought that starting a discussion about the teaching approach was too early. So I tried to capture their early reactions in my diary which reads:

"Students were assigned to work in groups but they were not working as a group. Instead they appeared to be working individually and competitively. They did not appear to be explaining their solution method to one other."

"The members of two groups appeared to be in disagreement within their group concerning the solution method, but without providing arguments. Their reasons were restricted only to a superficial level. They solved the problem following different methods, but were quick to blame each other for 'incorrect' solutions."

"After reinforcement and encouragement, the group members realised that for the purposes of presenting their work to the rest of the class they needed to have a common solution and explanation."

"The group members started to discuss the situation and decided as a unit about the solution method. They executed the calculations individually, and compared their results. The high-attainers appeared to take the leading roles."

At the beginning of the actual teaching stage in TE-1 the students worked in groups but not collaboratively. Each student concentrated on his/her own work. They could not 'see' any useful point in working collaboratively. Their indirect reactions to the new approach were obvious. If the students were not asked to work in groups they certainly did not take the initiative to do so. The competitive environment, to which they were accustomed, appeared to satisfy them during that time.

In a late session of the actual teaching stage, after we had finished the day's lesson, I asked students to comment about the teaching approach they involved in to solve the word problems. I also asked them to compare it with that of their teacher. The following is an extract from the discussion that took place:

I : Which way of solving problems do you like better?

Y : I like your way more.

I : Can you explain why?

Y : We explain the problems more.

M : With our teacher we explain only what we do not understand, while with you, we explain everything.

S : We know why we are using multiplication or division ...

I : Does this help you in any way?

T : It helps very much.

I : How?

G: We understand the problem better.

S: We know why we are using an operation.

...

F : Sir, can I say something?

I : Yes, Fotis.

F : I like both ways [yours and our teacher's] but I prefer to solve problems as we did together, working in groups.

I : Can you tell us why?

F : When you work in a group and you try to find the 'why' and all say their opinions you understand the problem better.

I : What do you mean?

F : When you have not grasped something in the problem, it might be explained by another group member and this might help you to understand it better.

A : It helps you to 'see' how each member in the group views the problem. I like to hear the other students' thinking processes.

...

K : Sir?

I : Yes, Koula.

K : We like to work in groups but we do not want Vasilis in our group.

I : I think we arranged this. Didn't we?

K : Yes. We like it better to work ...

...

I : Any other comments?

L : I like to work in groups and listen to other students' opinions.
They may be different from yours ...

S : When you work with other students sometimes you find more solutions.

I : How many groups agree with Stavros' claim?
[The members in three of the four groups agreed with that claim and they also said that they had such experiences.]

Although the students' responses in the above discussion may be insincere and not reflecting their true views about my teaching approach some points are of special interest. The comments made particularly by Fotis and Stavros I think are important. One can make such claims when one has similar personal experiences. I think that my actual teaching provided the students opportunities from which such claims could be made. I believe it is difficult for youngsters at this age to draw such conclusions if they lack relevant personal experiences. When the students were discussing my teaching approach they appeared very careful with their comments. Even though they seemed to prefer the experimental approach they did not want to be too critical of their teacher's method. Their answers appeared to be balanced and when they said something in favour of my approach, they indirectly implied that their teacher's method was almost equally acceptable (i.e. Y : I like your way more., F : I like both ways ...).

3.4.3 The teacher's role

When children are collaborating in a group to establish a way of carrying out a problematic situation, inevitably their way of working and their attitudes to word problems are vital. However the teacher's role appears to be important in the management of group tasks, in giving children the opportunity to think and in building a framework of communication.

3.4.3.1 Management of group task

I viewed my management of the group task as vitally important, especially at the initial phase of the actual teaching stage. I tried to minimise the risks to individual pupil's self-esteem by refraining from evaluative feed-back about their collaboration. When making comments, I directed them to the whole class, rather than to the individual or group. When observing or listening to the groups, I specified that I was not going to participate in their discussion. I was interested in their way of approaching the solution and I placed a great deal of emphasis on the importance of groups talking together, listening to each other, and working collaboratively. Giving the groups full control of their solution methods and allowing them privacy to discuss them was important. Initially, I would not publicly identify solution methods as belonging to a particular individual or group. As work continued, I felt I could ask for volunteers to explain their method of group solution. Eventually, I felt I could ask each individual in turn, in these whole class discussions, for his/her solution method. This worked well as I was able to protect the less confident members of the class initially, until they had successfully understood the solution method to the point where they could risk their contribution to the whole class.

Galton and Williamson (1992) justified collaborative group work on a number of grounds, which included the improvement of understanding among slow learners, the development of teaching skills among their peers, and the general development of social cohesion and collaboration within a group of children. Attempting to get collaborative group work started in my classroom, it was rather like starting a cold engine on a frosty morning. The challenge is to get the engine started, warmed up and running smoothly before the battery goes flat. My experience was rather like this, persevering with the initial hiccups and pupil reactions until a smooth running and effective learning sequence had been achieved. Here are some of the diary entries relating to the initial 'breaking in' period:

"In one group, the boys had moved and worked apart from the girls. In Elias' group, Tasos and Costas are not working well together and are annoying each other. Elias is trying desperately to manage and keep the group together. He mainly blames Tasos ... In Yiannis' group Georgia complains about the presence of Vasilis. He seems not be accepted by any group. There are clear tensions in many of the groups."

I tried not to divert from my original plan, and in some instances I was successful, as is shown by this extract:

"Since Vasilis could not cope with the Yiannis' group I moved him to Anna's group. Vasilis now starts to bother Anna. Anna leaves the group because Vasilis was teasing her and bursts into tears. I allow her time to reflect, she is offered no alternatives from me and returns to the group no longer feeling upset."

On other occasions, this strategy was not so successful:

"Anna asked to move from her group or to move Vasilis away. I encouraged her back, Vasilis insulted her, she reciprocated, and bad feeling ensued. I spent seven minutes sorting it out. This could have been avoided if I had instinctively moved them ... I tried to hold the groups together by intervening and managing disputes, rather than allow them to break-up into subgroups or individuals (Move Vasilis from Anna's group next session)."

I was faced with several teaching dilemmas, concerning mainly the case of Vasilis. He had been rejected already by two groups and was not willing to make an effort to follow another. He did not seem to have a close friend in the class with whom he would like to collaborate. Eventually, in order to let the groups work more smoothly, I asked him to come to my desk and work together with me. He accepted that while, at the same time, the groups appeared 'relieved'.

I do feel that trying out new ways of working and re-sorting individuals to work with others whom they do not choose, inevitably will promote a few initial problems of adjustment. It is sometimes important for the teacher to realise this, as often they revert to tried and tested techniques. This attitude is very understandable as from the teacher's point of view, setting up a new approach to teaching word problem solving is a daunting task owing to the lack of control. To develop and improve the repertoire of teaching methods, I believe risk taking is essential.

3.4.3.2 Class discussions

Throughout the teaching experiment, I held classroom discussions to give the children opportunities to reflect on what their group had been doing during the course of the problem solving. I allowed them an open forum in which they could explain their method of solution and compare this with those of other groups. At this point I would encourage the groups to express their solution methods and provide an ethos within which all solution methods were welcome. Through these discussions, I attempted to encourage children to say not only what operations they used but also to explain the legitimacy of their selections. It was important in these discussions for me to be non-judgmental. I tried not to offer my solution method, but to develop the discussion as a forum for the pupils' ideas on solution methods. I tried to avoid in these debates the form of discourse most often used by teachers and pupils in whole class discussions, in which a question and answer sequence organises talk into two-part exchanges, focusing attention on what has been said instead of encouraging listeners to encompass larger stretches of dialogue. This is the notion of 'cued elicitation' (Edwards and Mercer, 1987, p. 142) and it is discussed in section 4.1.2 In that respect it actually discourages the unquestioning pupils from engaging in reflective thought, and from making explicit their considered reasoning. Other researchers have also reported the 'cue-based behaviour' (Boaler, 1997, p. 37). However, I tried to avoid this in order to promote a higher order cognitive activity. The following episode illustrates my points.

Students were asked to solve the problem: 'The school's committee bought 150 books for 1.800 drachmas each. How many drachmas did they pay?' (MNER, 1993a, p. 52). After the 'what' and 'why' questions were answered appropriately and the correct answer was found, the students were asked, while still in their groups, to make an inverse problem. When I asked students to present their inverse problem the following discussion took place:

A : [She reads] : The school's committee bought 150 books and paid 270.000 drachmas. What was the cost of each book?
[As the student read the problem I wrote it on the board.]
S : Sir, we made a different inverse problem.
I : Would you like to share it with us?
S : The school's committee bought books for 1.800 drachmas each and paid 270.000 drachmas. How many books did they pay for?
[Again, I wrote the problem on the board.]

While the two problems were written on the board, Dina (classified in the low category during the pre-test and pre-interview) whose group was not very successful in developing an inverse problem, asked:

D : Sir, which of the two is correct?

A discussion began on the correctness of the two inverse problems. Although most of the students argued that both problems were acceptable, they could not justify their opinions. However, the discussion continued and after some time two high-attainers, Yiannis and Elias, came up with sophisticated answers. What follows is an extract from this discussion:

I : Who would like to answer the question?
K : I think that both problems are correct.
I : Can you explain why?
K : [no response]

...

Y : Both [problems] are correct. We can make an inverse problem using the 150 books and the 270.000 drachmas and we can make another inverse problem using the 1.800 drachmas and the 270.000 drachmas.

E : Yes. Since in the original problem two knowns and one unknown were given, we had two knowns to make them unknown. So we formed two inverse problems.

I : Is this clear to everyone?

...

I : Can we make a third inverse problem from our original problem?

M: Yes, we can.

L : No, no, we cannot.

M: Why?

I : Can you explain to us why we cannot?

L : Since we do not have enough numbers.

I : Can you explain a little further?

L : We should have another number ... [pause] another known for making a third problem. If we had three knowns we could form three inverse problems.

Discussions normally took place after the children had been involved in the problem solving situation and they had finished with their solution methods. Furthermore, discussions could also be initiated based on my own instinct, when I felt that the whole class needed some further explanation or clarification. The students became accustomed to this procedure. They were encouraged then to perceive when the moment was necessary to stop what they were doing and do some thinking. I am not sure how this might occur other than by chance, without it being teacher initiated. It often only took a question to help groups reconsider the current steps in their solution method. There was no evidence to suggest that this discussion was happening. Indeed my method of research would have needed considerable modification to pick this up by anything other than chance.

3.4.3.3 The value of building a framework of communication

Edwards and Mercer (1987) draw upon the basic ideas of Bruner and Vygotsky to suggest a way forward in the primary classroom which draws upon cognitive psychology for a set of basic principles. They are critical of Piaget's 'discovery learning' and suggest that children need much more. A major issue for Edwards and Mercer is the tension that lies between the demands on the one hand, inducing children into an established, ready-made culture and, on the other hand, developing creative and autonomous participants in a culture that is not ready made but is continuously in the making. Griffin and Cole (1984) emphasise the communal basis of knowledge as a future orientated process of culture and education: 'social organization and leading activities provide a gap within which the child can develop novel creative analyses ... a Zo-ped (zone of proximal development) is a dialogue between the child and his future; it is not a dialogue between the child and an adult's past' (p. 62).

Edwards and Mercer suggest education is a social process where knowledge is open to scrutiny. This scrutiny is a social process, not merely one of individual discovery, but one of sharing, contrasting and arguing ones perspectives against those of others. It is in this way that the collaborative nature of group work and whole class discussions provided the forum in which this could take place. Establishing and maintaining the quality of pupil talk and classroom discussion was essential. Clearly mixed ability grouping did enhance the self-confidence of two of the low-attainers, Costas and Dina, and enabled them to participate in the discussion with others in their group.

From a psychological viewpoint it would seem that learning failures are not necessarily attributable to individual children or teachers, but inadequacies of the referential frameworks within which education takes place. In other words they are what Edwards and Mercer (1987) describe as 'failures of context'. Good teaching needs to be reflective, sensitive to the possibility of different kinds of understanding. Pursuing good teaching, then, may involve the careful creation of context, a framework for shared understanding with children based on a joint knowledge and action which provides its own rationale for present activity and a strong foundation for future developments. This provides 'the 'scaffolding' for children's mental explorations, a cognitive climbing-frame - built by children with their Vygotskian teacher - which structures the activity more systematically than the discovery sandpit of the Piagetian classroom' (Edwards and Mercer, 1987, p. 167).

A communicative framework, involving talk between teachers and children helps build the 'scaffolding'. Teachers who retain tight control, dominating the agenda and discussion, determining in advance what should happen and what should be discovered, will find even their most successful pupils scaffolded like some supported structure. Here they will be unable to function independently or outside the precise context and content of what is done in the classroom. To make scaffolded learning successful it is important to emphasise the socio-cultural and discursive basis of knowledge and learning. I support the view that knowledge and thought are intrinsically social and cultural phenomena, and that students' learning is constructed through social relations and interactions. Linked strongly to this is the explicit communication which outlines to pupils what they are doing, and where it fits into what they have already done. The students' problem solving experiences in the classroom are made meaningful by the sense of their solution methods during the classroom discussion. To avoid offering students help in making sense of their problem solving experiences, will have the consequences that the usefulness of these experiences is lost, or that teacher and pupils resort to conventional means of communicating.

Collaborative group work and the process in which I engaged in this study emphasised the importance of communication in trying to help make the classroom experience more open and explicit to the pupils involved. I hope I went some way in developing a scaffolding for learning which lowered some of the barriers to learning as outlined above. Edwards and Mercer summarise this well, when they suggest that any approach 'which overemphasizes the individual at the expense of the social, which undervalues talk as a tool for discovery, and which discourages teachers from making explicit to children the purposes of educational activities and the criteria for success' (p. 170) can no longer be trusted. Certainly for teaching word problem solving in (primary) mathematics this alternative psychological approach appeared to work in this classroom. I am not sure to what extent it could be transferred to other areas of the curriculum.

3.5 Changes in students' attainment and behaviour

3.5.1 Students' attainment and behaviour in the pre-test and pre-interview

Through examining students' test papers for the pre-test, I gained an understanding of their learning, which appeared rather limited and incomplete with respect to the data I collected from the interviews. Listening to students explaining their solution methods, describing their thinking processes, clarifying points unclear to me and answering my 'why' type questions helped me to formulate a more complete picture of their mathematical knowledge in relation to the three word problems.

My general impression from the interview was that students seemed somehow surprised when I asked them to tell me how they thought about the problem. Most of the students who solved at least one of the problems, could specify very easily the operation/s they used, but they had difficulties explaining why. Before starting their explanation they had to stop and think for some time. In some cases I tended to believe that students although performing an operation had not thought about it before and when I was asking "Why ... ?" they had to invent some justification. This was more evident in the case of students who followed the 'wild goose chase' method (this is explained later in this section).

In relation to the solution methods they provided, students were classified in three categories: (i) the high category, (ii) the medium category and (iii) the low category. This distinction was made for the purposes of this research and without intent to permanently label students in these categories. The high category appeared to have the smallest number of students (4), the medium category the largest (6), and the low category an intermediate number of students (5).

The high category consisted of students who provided appropriate explanations. They not only described their solution methods but also justified them. Students in this category showed a high degree of understanding. They had meanings for the operations and they used these meanings to model the settings of the problems. They were willing to answer any question. They treated all the questions similarly and seemed as if they had the answers ready. There was no pause to think about the questions. They appeared confident and relaxed during the interview. Most of the students in this category solved all the problems correctly.

The second category, the medium, consisted of the students who solved one or two problems correctly, but could not explain their solution methods. These students offered mostly practical explanations and oversimplified their justifications. Mathematical arguments were non-existent. When I was asking "why ... ?" they were looking for an answer. Using their hand movements (more) and their facial movements they were indirectly trying to indicate to me that the answer to my ('why') question was obvious.

The students in this category could specify the operations in the problems but they could not explain their relevance in these particular problems. My interviews with these students did not suggest that they had concepts of the operations or if they used them as the basis for the operations they chose. I had the feeling that the 'what' type questions were easier for them than the 'why' type and also that they would prefer the interview not to include the latter type of questions. They needed enough time to think for supplying their explanations. During the interview they seemed a little nervous and anxious especially when they heard me posing the question "why ... ?"

The last category, the low one, consisted of those students who did not solve any of the problems correctly and did not demonstrate any understanding. Their explanations were irrelevant and often they were not responsive. Also included in this group is the student who declined to be interviewed.

The pupils in this category seemed to lack mathematical knowledge of the four basic operations. They went through the interview repeating themselves. It is characteristic that most of these students did not participate in the lessons in the preliminary observations, while in the interview which was a one-to-one basis, they felt more comfortable in exposing their thinking. In other words it seemed that some of these students lacked the confidence to talk in front of the whole class.

Students who did not solve any of the problems were asked if they had tried to, and what kind of difficulties had they encountered. These students reported that their difficulties were associated with the phraseology of the problems. One of them commented that he got confused with the words of the problem and he did not understand what the words 'said'. Another said that he could have understood the first problem had there been more words, and also said he did not understand the word 'collected'.

Other outcomes

- As I mentioned above one of the students was unwilling to come into the interview for explaining his solution methods. When I tried to approach him, he indirectly implied that the way one thinks is a personal matter.

- In the first problem, which was a straightforward, routine multiplication problem, three students had originally started it using division. Two of them gave me explanations for using a division which were based on a 'recency phenomenon' (Schoenfeld, 1985, p. 367). Their last lesson was on division problems and therefore using a division was an almost automatic response. Finally these two students correctly solved the problem.

- There were three students who used the 'wild goose chase' method. They performed operations without been able to explain why. One of these students used this method more often than the other two, but still managed to solve the problems. This particular student was choosing operations at random but was able to evaluate the reasonableness of the answer/s. In the light of an unreasonable answer, the student would revise the choice of operation and select another one. In contrast the other two students were performing operations without evaluating their results. However, the 'wild goose chase' method proved to be productive only for the pupil who had an awareness of the answer. For the other two the method was not effective at all. It was simply pointless.

- When I asked students to describe their thinking processes, especially in problems without appropriate solutions, they soon discovered their mistakes. So, when revising their thinking, they could figure out the correct solution. In fact, this happened in three cases, with three different students, without any prompting from me. Asking them to (re)think the solution method they had followed while working through a problem was an effective way of self-correction. Students were able not only to point out their mistakes and give reasons for them, but also could specify the correct operations.

- Four students working the second problem made the same mistake and realised it during the interview. Misled by the word 'altogether', they performed an addition, ignoring the rest of the data given in the problem. It is worth mentioning here that during my twelve preliminary observation periods in grade-four, I noticed that when the teacher had students on the board solving problems and they were having difficulties, she tried to help them by pointing out the key words. This particular teacher used this method as an established way of providing help to students who had arrived at a mental blockage. It appeared in the interview that the teacher's actions had an influence on the behaviour of these four students.

- The interview revealed also some difficulties students had with the questions in the third problem. This problem had two questions which were deliberately not separated with letters or numbers but were in sequence one after the other. Four students reported difficulties with the two questions. They said that they did not understand where they had to start. These students' explanations showed that they had not grasped the situation described in the problem. One of them mentioned that if the questions had been numbered, the problem would be much easier.

3.5.2 Students' attainment and behaviour in the post-test and post-interview

The post-interview appeared to present a different picture than the pre-interview. Students' experience from the pre-interview allowed them to feel more comfortable and relaxed as they knew what the interview would be about. In this interview, the student who had declined to attend the pre-test interview, was included. In fact, all the students were excited about the interview and they wanted to participate.

Listening to students explain and justify their solution methods gave me the opportunity to make important observations concerning their problem solving behaviour. I was particularly interested in finding out how they treated the word problems and how they approached the solutions.

The three categories of students who had been identified during the pre-interview high, medium, and low were still found to exist but in different proportions. (Rearrangement of students from the lower categories to the higher took place.) During the interview I noticed that, after the actual teaching stage, students' behaviour in problem solving appeared to have changed. All the categories showed an overall gain from pre-test to post-test. However, I should point out that pupils being in a higher category, does not necessarily mean that there is a change in their behaviour because the categories were defined with respect to different questions and hence they were not equivalent.

The first category appeared to have the largest number of students (8), the second an intermediate number (4), and the third the smallest number of students (2). The student who left the school was classified in the medium category during the pre-interview. The students' performance on the pre-test and the post-test, and the number of students performed high - medium - low on each test are shown in the Figure 3.3 and in the Table 3.3 respectively.

Figure 3.3 Students' performance on the pre-test and the post-test

Student's name	Pre-test			Post-test		
	H	M	L	H	M	L
Yiannis	V			V		
Litsa	V			V		
Elias	V			V		
Anna	V			V		
Stavros		V		V		
Georgia		V		V		
Koula		V		V		
Fotis		V		V		
Matina*		V				
Tasos		V			V	
Petros			V		V	
Dina			V		V	
Costas			V		V	
Nikitas			V			V
Vasilis			V			V

* She did not take the post-test because she moved to Greece.

Table 3.3 Number of students who performed high - medium - low on the pre-test and the post-test

Test	Students' performance level (number of students)		
	High	Medium	Low
Pre-test	4	6	5
Post-test	8	4	2

In the high category were classified four students who appeared to belong to this category during the pre-interview, as well as four other students who at that time had been classified in the medium category. These four new youngsters changed place and they moved from the medium category to the high category, as they provided appropriate explanations and justifications for the solutions they produced. Those included in this category explained their solution methods and justified them. Regardless of the correct execution of a mathematical computation, they had meanings for the operations i.e. using multiplication when several sets of the same number are put together.

In the medium category four students were classified during the post-interview. Only one of these belonged to this category from the pre-interview, while the other three students moved upwards from the low category. The students into this category could specify the (appropriate) mathematical operations in a problem, but could not explain them. Providing an explanation for their solution method did not seem to be an easy activity for the students within this group.

The last category, the low, contained the smaller number of students (2). In this category five students were classified during the pre-interview but now there remained two as the other three showed improvement and were moved to the medium category. These two students during the actual teaching stage, as well as in the post-test and the following interview, did not demonstrate any understanding. They attempted the problems, but could solve none of them. They selected operations without being able to explain why. They could not even explain their answers. It seemed that they did not realise what the problems were about.

There is a further important result of this post-interview. I never noticed any student to use the 'wild goose chase' method. Even the three who had mainly used this method during the pre-test had now abandoned it. One of these in particular who used it extensively and effectively showed improvement in problem solving, during my teaching experiment, moving from the medium category to the higher. The other two were the only ones remaining in their category (one in the medium, one in the low) in which they had been originally classified, but they preferred to use more stable solutions. However, after my actual teaching, their attitudes towards problem solving seemed to have changed. The random selection of operations had been abandoned. One of the three students characteristically claimed: "It is important when you use an operation to know why you use it." The operations were selected on the basis of the analysis of the situation described in the problem.

3.5.3 Implications for the next phase

There were two reasons why the results of the teaching experiment, with respect to the students' behaviour and their overall gain from pre-test to post-test, were not very convincing. Firstly, the pre-test and the post-test, with different problems and level of difficulty, were not comparable. Therefore I could not discover if students' change in behaviour was a consequence of my actual teaching or of other reasons (e.g. problems in the post-test).

Secondly, I had collected data of the whole class discussions, but not of the groups' discussions as an initial trial run of tape-recording had shown that the background noise and group movements had made the conversations inaudible. As a result I did not have enough information of the students' attitude and behaviour in their groups. This data would have helped me to explain their behaviour in the post-test and the following interview.

Therefore, the general implication of the teaching experiment-1 was that it would be better in the next experiment (i) to use the same assessment items (i.e. pre-test and post-test to be identical) and (ii) to establish a system of tape-recording small group discussion. Both aspects appeared to be important as they would allow me better to compare the students' attitude and behaviour before and after my actual teaching and explore possible changes.

CHAPTER 4 TEACHING EXPERIMENT-2 (TE-2)

4.1 Preliminary observations

Resultant upon the analysis of teaching experiment-1 I conducted a second series of preliminary observations in the Greek school between the 2nd and the 10th of April 1998. The new grade-four class was observed for six consecutive maths periods. At the end of each period, an interview with the teacher was carried out. The interview was based on issues raised from the current day's lesson but also from that of the previous one. In this grade-four class there was a total of seven children, four boys and three girls. The teacher was also the principal of the school.

Before the first observation period I explained to the students the purpose of my presence and assured them that all information would be confidential. Although two students at first seemed uncomfortable with the idea, they eventually accepted it, acted naturally, and even offered to help set up the equipment (tape-recorder).

After the six observation periods it could be concluded that the teacher's style was what academic writing refers to as 'transmission style teaching'. This view assumes that 'the goal of instruction is to transmit knowledge to students' (Cobb, 1988, p. 87). Instruction is viewed as a matter of laying out the knowledge and skills to be acquired. Consequently learning is assumed to be a process of absorption or direct transfer of knowledge into the mind of learner.

Burkhardt (1988) refers to this as 'the standard teaching style' and he describes it as follows:

> 'Mostly we <u>explain</u> to students the skills and concepts we want them to grasp, <u>illustrate</u> these with some <u>examples</u>, and coach them while they

practice some very similar imitative exercises, to make sure that they have learnt the techniques. In coaching we usually check whether they are 'off course'; if so we explain again the correct method, perhaps using different words, and, if they still have difficulty, break the exercise down into smaller steps. The student's role is imitative' (p. 17).

Schoenfeld (1988) argues that

> 'despite recent movements towards co-operative group work and open-ended investigations, one envisions a typical mathematics classroom as follows: a teacher orchestrating what takes place, either presenting material, having individual students present and defend their work at the board, leading classroom discussion (perhaps as Socratic dialogue), or having students do seat-work - each by himself or herself, copying what takes place at the blackboard or working an assignment sheet without access to support materials (e.g., texts) or the support of others' (p. 15).

Thompson (1992) says by quoting Kuhs' and Ball's (1986) how mathematics should be taught. According to her, reviewing the literature in mathematics education, Kuhs and Ball (1986) identified four distinct teaching views:

(i) learner-focused,
(ii) content-focused with emphasis on conceptual understanding,
(iii) content-focused with an emphasis on performance, and
(iv) classroom-focused: teaching is based on knowledge about effective classrooms (Thompson, 1992, p. 136).

Particularly, the second and the third view are focused on the content and therefore imply a transmission of knowledge. However, the emphasis of contents are different in these two views.

In the second view, instruction makes the content the focus of classroom activity but the responsibility for understanding rests upon the individual learner. In the third view, the emphasis of mathematics teaching is on the performance of mathematical rules and procedures. The role of the teacher is to explain and illustrate the material using the 'standard style' (Burkhardt, 1988). Accordingly, Thompson states that the role of the students is to 'listen, participate in didactic interactions (for example, responding to teacher questions) and do exercises or problems using procedures that have been modelled by the teacher or text' (Kuhs and Ball, 1986 cited in Thompson, 1992, p. 136).

In a study (Askew et al., 1997) which explored effective teaching on numeracy (in primary schools in England), three orientations towards teaching mathematics emerged: (1) the connectionist, (2) the transmission, and (3) the discovery. Particularly, the transmission orientated teachers placed more emphasis on the ability to reproduce procedures and routines although their expositions often included conceptual explanations. They did not give much attention to developing reasoning about number. The methods that pupils had created themselves were little valued. More emphasis was placed on teaching rather than on learning.

As mentioned above, the orientation of the grade-four teacher seemed to be in line with the transmission style teaching. Below are extracts from the observation sessions and references from the interviews with the teacher. These will illustrate and justify the classification of the teacher's style of teaching.

At this point, I should make clear that in the following dialogue and interview extracts, the interviewer / researcher is identified by the single letter 'I', the regular teacher (principal) by the single letter 'P' and the students by the initials of their pseudonyms (Figure 4.1).

Figure 4.1 Identification letters for students, regular teacher, researcher

Students' pseudonyms	Identification letter		Identification letter
Maria	M	Students as a group	**S**
Takis	T		
Stella	S	Regular teacher (principal)	P
George	G		
Niki	N		
Argyris	A	Researcher	I
Christos	C		

4.1.1 Students presenting their work at the board

Students presenting and defending their work on the blackboard is seen by Schoenfeld (1988) to be one of the basic teaching actions in a typical mathematics classroom. At the beginning of the periods observed, the teacher corrected the students' previously assigned homework. The students approached his desk with their work. If a student failed to solve a problem correctly, the teacher asked him/her to stay behind and attempt to solve it on the board. During the interview, the teacher was questioned about his tactics:

I : I noticed that the students come to your desk in order to have their work corrected.

P: Yes.

I : Are you doing this systematically?

P: Of course. Every time.

I : I noticed yesterday that you asked a student to stay and work the problem on the board.

P: Whoever has not successfully solved the problem tries to solve it on the board.

I : If they haven't solved any of them?

P: If they haven't solved any, all are solved on the board. The child will not leave the school without having understood the basics of the problems.

I : Even if a problem has not been solved by someone, he/she comes and solves it on the board.

P: It's meaningless to have somebody else solve the problem, since the person having the difficulty may not understand it. He/she does it himself/herself in order to understand what is happening.

During the six observations, all students were asked to come to the board except Argyris. Although the teacher knew that Argyris had never solved or tried any of the problems he never asked him to come to the board. During an interview with the teacher, Argyris' progress was questioned as it was noticed that he was not participating in the lesson.

I : Does Argyris participate in the lesson?

P: Not at all. Argyris doesn't know the basics.

...

I : Why is that?

P: He hasn't learned anything.

I : How do you explain that?

P: He is not interested, he is not interested at all.

I : Are his parents aware?

P: They know it. Look at his report card. [The teacher opens his desk's drawer and shows me Argyris' report card.]
Argyris is going from bad to worse, from bad to worse.

The teacher further explained that he had written three letters and phoned Argyris' parents twice so that they could come to the school and discuss Argyris' progress. So far there had been no response. He thought it important that parents take their child's progress in school seriously. Furthermore, that parents should be more sensitive to the school's invitations to discuss their child's progress. They should realise that school is not simply for 'moving' pupils from one class on to another.

As with the previous teacher, every time the teacher asked for volunteers to come up to the board, there was a great willingness on the part of the students. The only exception was Argyris. I never saw him raise his hand. This was discussed with the teacher in an interview.

I : I noticed that when you ask for volunteers to come up to the board most of the students want to participate.

P: Yes, they simply consider the board as fun. Many times they come even if they do not know the answer. They are asking for the board.

I : Why?

P: I think it is a reaction to this 'blind guide'. [The teacher refers to the textbook.]

I : What do you mean?

P: All the problems in the textbook are mechanical. They don't put the students in a problematic situation. These problems take the investigation of cause away from the students. Why do I do this? Many times the students solve a problem and they do not understand what they have solved.

When the teacher had asked someone to come up to the board he was working more with this student rather than with the rest of the class. In the interview I wanted to find out more about this teacher's behaviour:

I : I noticed that when you have someone on the board you work more with him/her. Am I wrong?

P: No, no. That's why he/she comes to the board. That's why an effort is made to come to the board as much as possible. But this doesn't mean that the rest of the students do not participate.

I : What do you mean?

P: When the student on the board finds difficulty, I am not telling him/her what has to be done, I ask the rest of the class and I talk last.

Although the teacher expressed this view, the transcriptions of the observation sessions indicate that his classroom practice was completely different. The interaction was limited to the teacher and the student who is at the board. The rest of the students did not seem to pay attention to the board but rather they appeared to be engaged in other activities. Interaction among students was non-existent in these observation periods.

4.1.2 Providing hints and using cued elicitations

Emphasising the required procedure for solving a problem rather than developing an understanding of it, is seen to be an approach used in transmission style teaching (Askew et al., 1997). During the observation periods, one common strategy the teacher employed in trying to help students in the solution process was 'cued elicitation' (Edwards and Mercer, 1987, p. 142). He used to repeat specific expressions of the problem's statement, or to emphasise (by intonation) these expressions, and in some cases to use both simultaneously. The following extract demonstrates the teacher's behaviour:

The students are working on a problem where a recipe is given for making a cake with eight eggs and other ingredients. They are asked to find the quantities of the ingredients that will be used for making the same cake, but this time with sixteen eggs.

P: Now the children want to make the same cake again, but what do they change?

C: Double. (*)

P: What do they change? How many eggs do they use the first time?

S: Sixteen.

P: How many eggs did they use in the original recipe?

S: Eight.

P: How many do they use now?

S: Sixteen.

P: What are the [quantities of the] other ingredients? (**)

S: [no response]

P: What is the relation between eight and sixteen? (***)

S: [no response]

P: It is double. Since they use double eggs what else should they do to make the same cake?

M: Double.

P: You should double all the other ingredients. Do the calculations. Do you understand?

In the above extract it is noticed that although the student answers the question (*), he is ignored. The teacher asks a question (**) and in the students' silence he reformulates the question more explicitly (***) making a lower demand on the students. Since the students still keep silent, it is the teacher who, in the end, presents the mathematical relation as noted also by Bauersfeld (1988).

At other times the teacher provided students with explicit hints even before they actively engaged in the activity and met any difficulty. For example, when the students finished the first graph problem and were ready to start the second one, the teacher commented:

P: Let's go now to see the second problem. Be careful. It is very similar to the first one. What do we know here?

During a following up interview the teacher was questioned about his tactics:

I : Don't you think that by asking students to find the relation between eight and sixteen you direct their thinking towards the solution?

P: Yes, in some way you direct them but you ask them to think as well. When you say to them, "Try to find the relation between eight and sixteen", you give them the end of the thread, but if they don't try to figure out that it is double, they cannot do anything by themselves.

I : What do you think would happen if you didn't ask them to find the relation between these two numbers?

P: They would try hard for another half an hour.

I : Would they find the answer in the end?

P: They would eventually.

The teacher's response implies that if the students were not provided with some kind of explicit help in the problem they would try hard for some time. Further he seems to believe that the students would finally solve the problem. He appears clearly aware of the students' abilities to solve a problem by themselves but he did not instigate any teaching activity that would facilitate such a learning approach in his class.

In another case the teacher asked a girl who had not solved a problem to solve it on the board. He read the problem. 'A farmer paid 212.500 drachmas to buy fertiliser. The price of one kilo was 85 drachmas. How many kilos of fertiliser did he buy?' In this situation the teacher repeated the expression 'the price of one kilo was 85 drachmas', adding a distinct stress to the word 'one', more than six times. The teacher was questioned about this:

I : I noticed that in the problem with the 'farmer' you used the expression 'the price of one kilo was 85 drachmas' many times. What was the purpose of doing this?

P: Because it was in the problem's statement, and the expression 'the price of one kilo was 85 drachmas' could help the student understand which operation she had to perform. That is why we were saying 'the price of one kilo was 85 drachmas.'

...

I : Do you think that the emphatic tone of your voice may indirectly indicate students the necessary operation?

P: Of course, many times, many times.

...

I : Why are you using these approaches?

P: It is in the process to try somehow to push students towards the correct method. Of course this implies that you will do it the first time or the second time, to lead them in the right way of thinking. Then they should undertake the responsibility themselves.

The teacher recognises that repeating and/or emphasising specific expressions of the problem may indicate to the students the appropriate operation. The transcription of the observation tapes provide evidence that the teacher used this strategy on a regular basis. However, the teacher's comments about "the correct method" and "the right way of thinking" are interesting. According to this view, there is only one acceptable way of thinking and therefore one correct method of solution, the one that teacher has in his mind.

> 'The desire to 'keep it going', to keep the lesson smooth and trouble free, works essentially against the main aims of the mathematics syllabus. The covert social structure of the classroom actions masks or supersedes the mathematical structures, which the teacher has in his mind and which he has tried to stage, and which the student can construct only through regularities of his own (internal and external) actions. In these situations, the learner's adaptive efforts towards an acceptable use of mathematical symbols and language are bound to generate context- and problem-specific routines and skills rather than insight, self-confidence, flexible strategies, and autonomy' (Bauersfeld, 1988, p. 37-38).

4.1.3 Providing verbal explanations

Providing verbal explanations to students is a basic element of the transmission style teaching (Burkhardt, 1988). Since this style places more emphasis on teaching rather than learning, it 'is believed to be most effective when it is consisted of clear verbal explanations' (Askew et al., 1997, p. 29). In all maths sessions the teacher had a tendency to provide verbal explanations to the students. The following extract demonstrates the teacher's behaviour:

The teacher had asked Stella to come up to the board to solve the problem: '1/5 of what number is 4?', and the discussion went as follows:

S: I didn't understand it.
P: You are given the number 4. This is 1/5 of what number?
S: 1/5
T: Oh! I understood it.
P: Can you tell us?
T: 4 is 1/5 of 40.
P: Before, in order to find 1/5 of an integer (whole unit), what did we do? We divided. In order to find 1/5 of an integer we divided the integer by 5. Now we know 1/5 of an integer, how shall we find the integer?
M: We shall multiply.
P: We shall multiply. Did you understand it?
T: No.
G: Say yes, Takis.
T: I didn't understand it. Why should I say yes?
P: If we had the number 20 and we were looking for 1/5 of it, what would we do?
T: 20/5
P: What would we find?
T: 4.
P: Now we know that 1/5 of a number is 4, what is the integer?
T: 20
P: How did you get it?
T: 5 times 4.

I also noticed that sometimes the teacher used to start an explanation and then leave it half way through so that the students could complete it. Below are two extracts, from a lesson about fraction units, where the teacher used both cued elicitation and partially verbal explanations.

Extract 1

P: What do we notice in fraction units?

T: It counts if the pieces are bigger.

P: If the denominator increases ... [The teacher expects students to complete the answer.]

S: Smaller.

P: The fraction unit has a smaller value.

Extract 2

P: The denominator shows us in how many pieces we have divided the whole unit. The numerator shows us ... [The teacher expects students to complete the answer.]

G: That it was one.

P: The numerator shows us how many pieces ... [The teacher expects students to complete the answer.]

G: [We] have.

P: We have. For example, if I say that George ate 1/2 of an apple, how much apple has he eaten?

In both extracts I noticed that the teacher had started an explanation ('question') and he expected students to complete it. When the students found difficulties the teacher provided more information (i.e. in extract 2, adding the expression 'how many pieces'). The students were 'expected simply to fill in the void at the end of the question as in a completion test' (see Bauersfeld, 1988, p. 35).

At the end of the next day's observation, the above interaction between the students and the teacher was discussed. The interview began with the discussion of extract 1.

I : I noticed that sometimes you start an explanation and then you leave it half way through. Am I wrong?

P: You are right. I try to understand if they have grasped the meaning, and when I see that they have, I talk, because I want them to learn it with the correct terminology.

I : Can you be more specific?

P: I don't want them to start saying 'smaller pieces', 'smaller cakes'. Because I have noticed that in mathematics particularly, the way you express a thought for the first time, will accompany you for all your life. Last year they learned fractions with cakes and this year they talk in cakes. So what is the problem? It will be necessary [in future] to move from cakes to the abstract notion of fractions.

...

I : In extract 2, I think there is no terminology.

P: There is no terminology. It simply is the same as if I were asking them 'What does the numerator show us?'. The interaction is more direct.

I : After, you added the expression 'how many pieces'.

P: Yes, I wanted to help them.

Although, generally, the teacher appeared very frank and fluent and during this interview did not have any difficulty commenting on extract 1, he did appear to be a little uncomfortable with the questioning of the second extract. I recognised this and did not continue it any further.

4.1.4 Limiting interaction to question and answer

During classroom instruction the teacher often addressed questions to students and expected them to answer immediately. If the questions were not answered, he supplied the answer or repeated the question in a more explicit and direct way. The students were rarely given time to think about the question. When they did, they were interrupted by the teacher's answer. One example was when the teacher asked a student to come to the board to solve a problem:

P: What do you know in the problem?

M: [No time to respond.]

P: You know how much he paid for the spaghetti and how many kilos he bought.

What does the problem ask you to find?

M: [No time to respond.]

P: How many drachmas per kilo he paid for the spaghetti.

Other times, the teacher repeated the question and directed the students more towards the required answer:

P: One kilo costs 10 drachmas. How much do the two kilos cost?

S: 20 drachmas.

P: What is the relation between 1 and 2?

S: [No time to respond.]

P: The kilos are doubled. What happen to the drachmas as well?

In another case the students were working on representing division with the terms: dividend, divisor, quotient, remainder.

P: What do we have to add here?

N: [No time to respond.]

P: The remainder. How should we write these two operations? [D=(qxd)+r]

How many operations do we have here?

Somehow we have to separate them. Don't we?

N: [no response]

After this classroom observation period, I discussed, during the interview with the teacher, his habit of answering his own questions. I would argue here that questioning a teacher about his/her teaching behaviours can at times be a delicate and risky task and therefore I approached the issue with caution and tact.

I : I noticed sometimes that you ask questions of your students and you may answer them yourself. Am I correct?

P: If they don't answer them! [Both of us laugh.]

Look, when you see that they are 'stuck' you try to find ways to help them.

I : What kind of ways?

P: Many times the only way to help them is to give them a start.

I : In what?

P: In the answer, or you may give them the whole answer to the question.

I : Do you think the students need more time to think about the question?

P: I don't know... [pause]

Other times they come to the board, they see a problem and their mind immediately grasps it. Other times they come and they have difficulty. That is a matter of attention, a matter of period, a matter of mood. All these interact together.

The teacher avoided answering the first question in a direct way. He mentioned "ways" of helping students but then he referred only to one way, that is the easy strategy of providing the answer. In the last and important question about the students' available time for thinking, the teacher answered "I don't know." This response prevented my further questioning.

It is important to point out here that as far as the strategies the teacher used to help students are concerned, there was not a particular pattern or tendency to use one strategy more than the other. Nor were they used in any particular way. Simply, the teacher had developed a repertoire of strategies for helping students and he used them alternately and in a variety of combinations.

4.1.5 Seeing less value in students' responses

Another important observation from this grade-four class concerned the teacher's attention to students' responses. Although students offered some interesting answers, these did not draw from the teacher a deeper interpretation. He appeared to look at students' responses superficially and only for the correct answer. I had the impression that his main aim was to lead the students towards the solution or the answer he had in mind rather than on developing an understanding. For example, the students were working on a graph problem, where the prices for both variables were given, and the teacher asked the students:

P: What conclusion can I draw by seeing this straight line?

S: I know how many kilos [She wanted to say drachmas.] one [kilo] costs.

P: No, I don't know how much the other costs. I know that the relation between these two variables is ... [He expects students to complete the answer.] What kind of relation did we say that it is?

A similar teaching behaviour was observed in many instances. In the cakes' problem for example, where the students now want to make a cake with 4 eggs, the following interaction took place:

P: Maria in the second case how many eggs do we use?

M: 4

P: What is the relation between 4 and 8?

M: 2 times 4 is equal to 8.

P: No, you said it inversed.

M: [no response]

P: The 4 in relation to 8, what is it? The 4 in relation to 8.

M: 8 divided by 4 is equal to 2.

In both cases the students' answers indicated that they have developed an understanding of the activity. But these answers did not get the teacher's special attention since they were different from the expected ones. Therefore, the ideas that students had themselves developed were not used as the basis from which to build a clearer understanding of the problem (Askew et al., 1997). In the first extract the student's [Stella's] response implied that by looking at the graph she could figure out the relation between kilos and drachmas. In the second extract the student's reply "2 times 4 is equal to 8" could have formed a good starting point for the expected correct answer. The teacher's avoidance of asking the students to explain their thinking further, gave me the impression that he was interested only in the right answer rather than if the students had developed an understanding of the situation. What the students knew was seen by the teacher to be of lesser value.

4.1.6 Emphasising mathematical terminology

The teacher's emphasis on mathematical terminology was another common observation in all maths observation sessions. The teacher seemed to be focused on the content of the lesson as well as on the terminology underlying that content. On most occasions the teacher interacted with students in mathematical language and they had difficulty in understanding him. Below are two extracts from two different lessons.

P: Okay, we have two variables. Which variables do we have?
S: The kilos and the drachmas.
P: The kilos and the drachmas. These two variables are in direct proportion.
S: That is?
P: That is, when the kilos are increasing what will happen?

In another case:
P: George do substitution now.
G: [No response. George is unable to follow the teacher's suggestion.]
P: Substitute the letters with numbers.
G: [Again, George cannot follow the teacher's instruction.]
P: For what number does the M [minuend] stand for?
G: 128

In the interview when discussing his emphasis on mathematical terminology, the teacher revealed some interesting views:

I : Do the students know the term 'substitution'?

P: They have been taught it.

...

I : I got the impression that you stress terminology.

P: Of course.

I : But does the book do that?

P: No, the book doesn't stress that the students learn the terminology.

I : Does this help them?

P: It is not a matter of whether it helps them.

I : Why are you doing this then?

P: I like my students to express themselves in a concise way, to use the right words and to say few things, because the more they talk the more mistakes they make. I prefer to introduce all the terminology and hope they will remember some of it. I have seen from my experience that from the 15 terms that students will hear, if they recall 3-4, this is enough. If you take it collectively, year by year, at the end of the primary school (in grade six) they will know all the important terminology. Even if sometimes they have not understood a hundred per cent what exactly a specific term means. I do the same thing in science.

In another observation period the students worked for one and a half periods on how the four basic operations are represented by the terms like product, sum, dividend etc. The students experienced a lot of difficulty with the exercises. By the end of the lesson the teacher appeared to be upset and exhausted. During the subsequent interview, the following interaction took place:

I : I noticed that when you explained to the students how we represent the operations with the terms like sum, divisor, product etc., you gave them some examples too. Why did you do that?

P: I wanted to help them understand the correspondence with letters.
Sometimes it is my fault, because I am asking too much from the students.
For example representation with letters is too much for them but they should learn to think.

It could be argued that if the teacher did not give students the examples they would not have been able to work on the exercises. The whole lesson would have been a 'disaster'. The teacher's view that he asks too much from his students is correct for two reasons. Firstly the book itself does not devote more than a lesson on the representation of each operation, with terms like product, quotient, sum etc. Secondly, the grade four students may have found it difficult and they may have needed specific numbers in order to work out a problem.

4.1.7 Presenting the content of the lesson

Although the teacher had the content of the lesson as the focus of classroom activity, he gave me the impression that he was not strictly adhering to the textbook. A variety of means were used to begin a lesson. Sometimes he started the lesson by oral discussion and expository teaching on the board (e.g. the graph problem), other times by using the book (e.g. representation of operations with terms like remainder, difference, product etc.), other times by oral discussion and giving students problems (e.g. 'fraction-units', paper to be cut in equal portions). Whenever the teacher thought it was necessary to take the initiative he did so without hesitation. When they started the exercises on 'fraction-units' in the book, he changed the order of the exercises for the students. The teacher was questioned about the change:

I : I noticed that you asked students to start with the second exercise and then to do the first.

P: Yes. In exercise 2 the fraction units are more 'clear' than in exercise 1.
The exercise 1 includes the strengthening from smaller to bigger.
Exercise 2 is easier as an exercise because it simply requires students to

visualise and write the fraction unit.

The teacher's answer implies that he is aware of the purpose and the degree of difficulty of each exercise. Further he seems to prefer the strategy of going from easy to difficult activities.

The teacher assigned exercises and word problems not included in the set textbook. He did that mainly during the week-end since on week-days the students had limited time available. In addition, as the teacher explained, in each class test (which was pre-determined and they may have known it) the students got an extra problem (unknown) from the teacher.

Although there were only seven students in the class, the environment appeared active and noisy but not annoying or troublesome. There was a friendly atmosphere in the classroom. The teacher had the habit of telling the students short jokes during the lesson and especially when they were making mistakes.

When some students (particularly the high-attainers) had difficulty, they asked their teacher directly for an explanation. Below are extracts of two such instances from two different observation periods. From the lesson on the graph problem:

P: These two arrows mean that the axes are infinite.
G: What does infinite mean sir?
P: Infinite means that the axes do not end. The numbers are infinite. In a similar way these variables and their relation are infinite.

From the lesson on representing the four basic operations with their terms:

N: [She writes on the board] : Subtrahend = Minuend + Difference
P: Do you agree?

S: I do not know because I did not understand.

T: Yes, sir. [He agrees with Stella that he has not understood it too.]

P: I will do an example. [The teacher writes on the board] : 4 - 1 = 3. Which number do I want to find?

It should be pointed out that only the high-attainers addressed questions to teacher. I never observed any of the medium-attainers or low-attainers asking anything in class. When they had any question they seemed to prefer to discuss it with the classmate who was sitting next to them.

4.1.8 The teacher's extrinsic rewards

Extrinsic rewards were not a dominant feature in the grade-four class. In the interview the teacher was questioned about rewards. He was quite clear.

I: Are you using reward expressions?

P: Only in the notebooks.

I : Yesterday I heard you in the class saying "bravo" two times.

P: Sometimes I use them in class only to the students who need encouragement, not to the rest.

The teacher's behaviour in relation to rewards is in line with the current research. In my work no rewards will be used. Rather it is hoped that the appropriate solution methods will be developed through class discussion. Damon and Phelps (1989) discourage the use of extrinsic rewards and they argue that small group learning of new knowledge and skills should provide its own intrinsic reward. In their view, it is a mistake to think that cooperative group work relies on extrinsic motivation rather than on the dynamics of social influence. An 'approach of encouraging students to talk about their solution methods without evaluating them for correctness is characterised by the development of a mutual trust between the teacher and the students' (Yackel et al., 1990, p. 20).

4.2 Design of instruments

4.2.1 Design of the pre-test and pre-interview

Prior to the beginning of the teaching experiment-2, a written problem solving test (Figure 4.2) was administered to all students (6) in the class. (One having returned permanently to Greece.) The objective of the test was two-fold. Firstly, to assess students' understanding of one-step word problems which had already been taught. Secondly, to assess their entering competence in multi-steps word problems which were going to be taught. The test included five word problems unfamiliar to the students and they were expected to complete all of them. The test did not aim to assess skill at mechanical computation.

In the first three word problems the solutions were presented in a multiple-choice format, and the children had to select the relevant mathematical expression (see Brown, 1985, Brown et al., 1984). The students were familiar with one-step problems but not in the format that was given. Word problems accompanied by their solutions in a multiple-choice format was something new to them, and they had to work the problems backwards (i.e. from a set of possible solutions they had to select the correct one).

The fourth word problem was three-step and the fifth a four-step problem. These two although they appeared to be routine, were problems the students had never before encountered. That is, the grade-four students had no experience in solving problems requiring more than one operation.

Prior to the administration of the test, a word problem (with a set of mathematical expressions as possible solutions) similar to the first three in the test, was practised on the board and discussed with students. It was important that they were aware of what they were expected to do in the first three word problems which they had never seen before.

Figure 4.2 Test given to students before and after the actual teaching

Pre-test / Post-test		
Class: D		
Name:..		
Problem 1		
312 photographs must be placed in an album.	312x12	12+12
On each page of the album should be placed	12/312	312-12
12 photographs.	12+312	12x26
	12-312	312/12
How do you work out how many pages are		
needed?		
Problem 2		
Everyday Pericles' father goes to work and	27x22	22-27
comes back with his car and covers 27 km.	22+27	27-22
	594/27	22/27
How many km does he cover in 22 days?	22x27	27+27
Problem 3		
A truck driver has to drive 520 km to get from	260x2	520+260
Athens to Salonica. After driving 260 km he stops	260/520	520-260
for lunch.	520x260	520/260
	260-520	260+520
How do you work out how far he still has to drive?		
Problem 4		
37 men and 23 women are working in a factory. How many hours of work do		
they complete together in a day (eight hours)?		
Problem 5		
The mother bought 10 cans of milk at 100 drachmas per can, 4 kilos of sugar		
at 150 drachmas per kilo and one kilo of cheese. She paid 3.600 drachmas		
for all these.		
How much did she pay for the cheese?		

Students were familiar with the format of the last two (i.e. Problem 4 and 5) and they knew how to work them. Thus there was no practice problem done on the board. Although the students considered the test to be rather easy, it was found that their performance was rather low.

After the students had written the test and completed the attitude questionnaire, interviews were conducted. These took place in the drama room of the Greek school, as in May of 1995 with the teaching experiment-1.

The interview dealt with the problems on the test; all students were asked to explain how they thought about these five problems. That is, students were expected to explain the solution method they followed in each problem on the test, and also to justify their choices of the operations they used. In addition where students gave interesting and unexpected responses on the attitude questionnaire, they were also asked questions about these.

4.2.2 Design of the attitude questionnaire

In teaching experiment-2, the students' motivational orientations were explored. The attitude questionnaire was developed to examine any changes in students' attitudes towards mathematics as a result of the teaching experiment. The students were asked to complete this questionnaire both before and after the experiment.

The attitude questionnaire developed was a result of combining the 'Personal Goals Scales' and 'Beliefs About the Reasons for Success Scales' developed by Nicholls et al. (1990). Some items were left out, others were combined and some were reformulated to fit the particular needs of the research. The students had to respond to the items on the questionnaire on a four-point scale under each question: 'Yes, yes, no, No' (i.e. strongly agree, agree, disagree, strongly disagree).

The items included in the attitude questionnaire, were presented in a mixed sequence (Appendix 5), and they were designed to assess students' views about the following motivational orientations (Cobb et al., 1992):

- Effort (working hard)
- Understand and Collaborate (making sense and collaborating)
- Ego (being superior to peers)
- Conform (conforming to teacher's or peers' solution methods)

Prior to the administration of the attitude questionnaire to the students, it was discussed with two other colleagues and piloted with four students from the grade-three class. As a result of piloting, three of the items on the questionnaire were reformulated. However when administered to the grade-four students, an unexpected procedural difficulty emerged. Two students on two items and one student on one item wanted to ring both 'yes' and 'no'. One explanation for this is that students had probably forgotten the stem ("I feel pleased in math when ... ") and they treated these items as independent, without reading the stem at the beginning of each item. This had implications on their responses as in some items their responses seemed to represent facts rather than opinions.

4.2.3 My understanding of students' perceptions and behaviour

My understanding of students' perceptions and behaviour was the result of three sources: (1) the preliminary observations, (2) the pre-test and (3) the pre-interview. I now describe my perception of each student in grade-four.

Maria

She sits in the first row of desks and is always concentrating on the lesson. She participates and she is interested in maths. During the interview, it seemed that she was aware of the operations and she was choosing the operation whose meaning fitted the story. She was the only one who solved all the problems correctly. She appeared to be the most able student in the grade-four class.

Takis

He participates in the lesson and he is always concentrating on it. Sometimes he makes mistakes because he wants to finish the activities first. In the interview he was able to explain and justify his solution methods as well as to find his mistakes. When he was questioned, he replied by stating the operation first. His performance on the test was above average.

Stella

She is a very shy girl. In fact this was discussed with the teacher and he agreed. She also sits in the first row together with Maria and Niki. She participates in the lesson when asked. During the interview, it was evident that although she had identified the necessary operations in the problems she could not justify them. However, she managed to solve two problems and half of another correctly.

George

He sits at the back of the class together with Argyris. He is a very active student with disruptive behaviour. He hardly ever concentrates on the lesson as he always finds something else to do. Although he is engaged in other activities he sometimes can cope with the rest of the class. During the interview a mixed picture of his understanding was formed. He solved one problem and half of another correctly but he justified only one.

Niki

Her participation in the lesson is very low. Even when she is asked, she can very rarely make a contribution to it. Her performance on the test was low. During the interview she did not seem to have meanings for the operations. The explanations she offered were oversimplified. She solved two problems correctly but she seemed to have great difficulty in justifying them.

Argyris

He seems to lack basic mathematical knowledge. His participation in the lesson is minimal. He does not pay attention and he does not seem to be interested in learning. He usually demands the teacher's attention in order to bring him back to the lesson. His performance on the test was the lowest. During the interview he could not explain anything. Together with George they are the most disruptive students in class.

4.3 Selection of groups

4.3.1 Establishing mixed ability groups and classroom organisation

The students' ability level was determined on the basis of the following criteria:

(I) the students' performance on the pre-test,
(ii) the outcome of the pre-interview,
(iii) the teacher's opinion about the students' ability
(iv) the review of past tests and
(v) the researcher's experience of the preliminary sessions.

When the teacher was questioned about the students' ability level (by the researcher), he gave a picture that seemed to agree with the students' performance on the past tests. He explained that there were in the class of grade-four, three high-attainers (Maria, Takis, Stella), one high-medium attainer (George), one medium-attainer (Niki) and one low-attainer (Argyris). However, the students' performance on the pre-test (Figure 4.3) and the following interview suggested a different picture for the students' ability level, that is, Maria was found to be high-attainer, Takis and Stella to be medium-attainers, George to be a medium-low attainer, and Niki and Argyris to be low-attainers.

Figure 4.3 Students' performance on the pre-test

	Maria	Takis	Stella	George	Niki	Argyris
Problem 1	v	x	x	x	v	x
Problem 2	v	v	v	v	v	x
Problem 3	v	v	x	x	x	x
Problem 4	v	v*	v*	v**	x	x
Problem 5	v	x	v	x	x	x

* They did not identify the last operation (addition).

** After the two multiplications he interpreted the results in an odd way without combining them.

Despite his average performance at the pre-test, Takis, during the preliminary observations was noticed to participate in the lesson and sometimes to compete with Maria, giving the researcher the impression of a high-attainer. The students were assigned a final ability level as described under the researcher's view in the following Figure (Figure 4.4).

Figure 4.4 Students' ability level

Students	Teacher's view	Pre-test / interview	Researcher's overall view
Maria	High	High	High
Takis	High	Medium	High
Stella	High	Medium	Medium
George	High-medium	Medium-low	Medium-low
Niki	Medium	Low	Low
Argyris	Low	Low	Low

As a result of the preceding analysis the students were assigned to work in two small groups of three students each. Each group was mixed sex and mixed ability, of narrow range (one group with high-medium attainers and one group with medium-low attainers). The reason for this decision was described in section 3.3.1 Group one consisted of two girls and one boy and had two high-attainers (Maria, Takis), and one medium-attainer (Stella). Group two consisted of two boys and one girl of which two were low-attainers (Niki, Argyris) and one was medium-low attainer (George).

The teaching experiment-2 was conducted over a two week period with five maths sessions in each week. During the first five sessions of teaching experiment-2 the students worked in the two mixed sex and mixed ability groups. After the fifth session, the students were merged into one group, and they remained in one group for the next five sessions of the teaching experiment-2.

Assigning students to work in two groups of mixed abilities and narrow range (one group with high-medium attainers, and one group with medium-low attainers), appeared to cause motivational problems to the second group (medium-low attainers). The students in this group did not complain about the mixed sex groups but did complain about the difference in the abilities of the students in each group. This problem was further emphasised during class discussion, when both groups had to explain and justify their solution method on the board. Then, the difference in ability between the two groups became more obvious. As a consequence of this, towards the end of the first week of teaching experiment-2, the students of group two, declined to come up to the board. However when in the second week the two groups were joined together, the students of the second group (medium - low attainers) appeared happier.

4.3.2 Explaining to the students the teaching approach

On the very first day of my actual teaching stage in TE-2, the group work was introduced to the students. It was important to do that as the students were accustomed to individual work and were encountering group work for the first time. Thus two posters (Figure 4.5) of different colours, with a list of group functioning guidelines written on them were affixed on the sides of the board. The first poster had five guidelines on it referring to the collaborative group work, and the second poster had two guidelines on it referring to the class discussion. I should point out here that in the teaching experiment-1 I used only one poster (the second one) for the expected behaviour. However, in teaching experiment-2 it seemed to be necessary to use two posters, one on the desk for the group work and another on the board for the class discussion.

I explained to the students how they had to work (i.e. collaboratively in mixed sex groups) and what sort of behaviours they were expected to demonstrate during group work (i.e. ask questions, answer questions, suggest solution method, understand solution) and class discussion (i.e. explain and justify solution method).

Figure 4.5 Posters with guidelines for group and class discussion

Poster 1	
On the desk (During collaborative group work)	
WORK TOGETHER with the other group members	
ASK for something you do not understand	
EXPLAIN your thinking and/or something you know	
SUGGEST a solution method for the problem	
UNDERSTAND the problem's solution	
Poster 2	
On the board (During class discussion)	
EXPLAIN	(What operation did you use?
and	
JUSTIFY	(Why did you use this operation?)
the problem's solution.	

The two posters which had the students' expected behaviours summarised in key verbs, and which were able to be seen easily, did seem to affect the students' behaviour. On three occasions the students (Maria, George, Takis) were observed to comment on these two posters. The first occasion observed was Maria, who had solved the problem with her group members. By pointing to each one of the five guidelines on the first poster she commented: "We did that (1st guideline), we did that (2nd guideline), we did that (3rd guideline), we did that (4th guideline), we did that (5th guideline)". On another occasion George said: "Sir, we are ready to come (to the blackboard). We did all these (the 5 guidelines on the first poster)". On the third occasion, Takis with his group members presented their correct solution method on the board. When they finished, he pointed to the second poster, commenting: "We explained what we did, and we justified why we did them".

I spent some time modelling the desired behaviour. With the students' assistance, a collaborative learning activity on solving a word problem was performed through a role play. I wrote a word problem on the board and I asked the students to work as a single group to solve the problem. When the students began to suggest operations and solution methods, I urged these students to explain why, or I encouraged other students to question the students who had provided the tentative solution methods. Playing the role of a group member and pretending that I had not grasped the problem, I asked the students (mainly the medium-low attainers) about the given and the unknown information of the problem. Through this modelling activity, I made an effort to demonstrate to students how to work in their group. Slavin (1992) explains that 'making the group goal and means of achieving it as clear as possible may also focus group members' efforts on effective helping' (p. 156).

I emphasised that as members of the same group they needed to work collaboratively, building on one another's strengths, rather than working competitively against one another's capabilities (Adger et al., 1995). I made clear to the students that there would be no intervening by me in their group discussion. Group members were to resolve any conflicts about the solution methods among themselves first and then discuss them with the whole class. Although my presence may have had an effect on the interaction between the students this was necessary in teaching experiment-2. The reason for this is that at the beginning of the actual teaching stage, when the students discussed a solution approach on the tape, they were identifying operations without mentioning the numbers. Therefore, during the collaborative group work, I observed each group at different times.

4.4 Analysis of the actual teaching stage

In this section, the findings of the actual teaching stage will be presented and discussed. These findings are organised into three core categories identified by the analytical procedures (see section 2.16):

- students' attitudes to word problems during group work (section 4.4.1)
- students' collaboration during group work (section 4.4.2)
- the group's influence on students' behaviour (section 4.4.3).

4.4.1 Students' attitudes to word problems during group work

This section points out how students approached solutions, how they verified the answers, how they saw their roles in problem solving, and finally how they coped with writing out the 'thinking procedure' and the explanation of the 'answer'.

4.4.1.1 Developing meaningful solutions versus producing answers

The transcription of tapes revealed that group one (the high-medium group) and group two (the medium-low group) differed in the way they determined the operations in a problem. The former appeared to be able to identify the required operations as well as to explain their reasoning. Each time they suggested an operation they provided a reason. In contrast, group two used to identify operations and perform calculations without justifying their reasoning. The choice of an operation was not followed by an explanation.

Below are examples of the two groups' approaches to the identification of the operations in the following problem (Unless stated otherwise the problems below have been developed by the researcher.) :

'Stella bought a book, 10 notebooks and a satchel. Altogether she paid 2.700 drachmas. The book cost 520 drachmas and each notebook cost 68 drachmas. How much did Stella's satchel cost?'

Group one

S: [The students read the problem.]

M: We shall do a multiplication, 10 times 68. (*)

T : Why?

M: To find out how much the 10 notebooks cost.

T: Okay. 10 times 68 is 680.

M: All right. The book cost 520. We will add them together, to find out how much the notebooks and the book cost. And then we shall subtract. (*)

S: To find out how much the satchel cost. (*)

T: Okay. First we will add 520 and 680 to find out how much these two items cost. (**)

M: And then we will subtract it [the sum] from the 2.700, to find out how much the satchel cost. (**)

T: Let's do it now.

S: [The students start executing the identified operations.]

Group two

S: [The students read the problem.]

A: [He starts to make a noise.]

N: I did not understand it.

G: Let me read it again.

A: [He bothers both George and Niki.]

S: [George and Niki ignore him.]

N: What are we going to do?

G: We shall add these two [520, 68] and then subtract them from 2.700.

N: So, first addition and then subtraction.

G: Yes.

N: Good, good, good.

S: [They start performing the addition 520+68.]

A: [He keeps making a noise.]

G: Niki, Niki?

N: What?

G: After the addition we shall multiply 588 by 10.

N: What did you say?

G: The addition is for one notebook. After we shall multiply it [588] by 10.

N: Can you say it again?

In the above extracts we noticed that group one and group two identified the operations to the problem in a different way. The students of group one read the problem, discussed it and made a solution plan. They identified the operations (*) and they also provided reasons for the operations. Before they started performing the calculations, they repeated their solution plan (**) within the context of the given numbers. Everyone in the group seemed to be aware of the necessary operations as well as of the required order.

In contrast, the students of group two identified the operations but did not justify them. In all the problems they worked together, these students (as is also discussed in the next section) tended to choose the operations without explaining why. Although George attempted to justify the multiplication (i.e. 588x10), his justification was rather inappropriate. Niki tries to challenge George by asking for repetition.

George's and Niki's behaviour does suggest that they may have misunderstood the nature of the activity, and therefore were not sure of what kind of behaviour was expected or that they may have found these problems challenging and they could not work out the operations. Furthermore, it was noticed that when George and Niki were moving in a wrong direction in their solution method, there was not anybody else in the group to challenge them. The special case of Argyris is discussed in section 4.4.3.4 below.

4.4.1.2 Verifying the answer versus evaluating the reasonableness of the answer

The two groups differed in the way they approached and verified their answer to the problem. The students of the first group were able to identify the operations for the problem, justify them, carry out the calculations, and finally to verify their answer. In contrast the students of group two worked backwards. An operation was evaluated by determining if it yielded a reasonable answer. In the light of an unreasonable answer another operation was used. The students' behaviour in the following problem illustrates the point:

'Mr Charlie and his son work together. Altogether they earn 9.000 drachmas per day. Mr Charlie in a week (5 days) earned 27.500 drachmas. What was his son's daily wage?'

Group one

After a discussion of an inappropriate solution method the students' interaction continued as follows:

M: I got it. We will divide 27.500 by 5 to find out the father's daily wage. Because it says that he gained 27.500 in 5 days. And then ... [pause]

T: Subtraction. To find out the son's.

M: Yes, subtraction. We will subtract what we get in the division from the 9.000, to find out how many drachmas his son gets per day. Did you understand it ?

T: Yes, yes Maria.

S: Yes.

S: [They start to perform the two operations 27.500/5 and 9.000-5.500.]

...

S: [After they have correctly performed the calculations in the two operations.]

S: So his son gets 3.500 drachmas per day.

M: And this is correct. Because if we add it with 5.500 we get 9.000 which is both their daily wage.

Group two

After an incorrect operation (27.500/9.000), the students performed the correct one (27.500/5=5.500) without being able explain what the 5.500 stood for. Their interaction continued as follows:

G: Now we should do 9.000/5.500.

N: Do you mean divide?

G: Yes. Because this [9.000] is how much they get in one day.

S: [They both perform the division 9.000/5.500.]

N: [She finished first and found 11.]

It cannot be. I have done it wrong.

G: [He found from the division 1.]

Niki, check if the multiplication [9.000x5] is <u>right</u>. If the multiplication is <u>right</u> we will cross out the division. We do not use the division.

N: That's better.

G: Yes, but we have to wait first to see if the multiplication is right.

S: [They start performing the multiplication 9.000x5.]

The above extracts demonstrate that each group had its own way of approaching the answer to the problem. Group one most of the times, had an outline of the solution. The students used this outline in order to arrive at the answer. Verifying the answer to a problem seemed to be a consequence of carrying out the operations which the students had previously identified and explained. The students of the first group appeared confident about the answer and they were able to verify it. In some cases they double-checked the answer.

However, a different behaviour was demonstrated by the students of the second group. An outline of the solution was existed but it was always rather incomplete. It could be used to start the solution but it was not enough to complete it. Additional operations could be chosen during the process of problem solving but they were chosen on the basis of the reasonableness of the answer. So the 'wild goose chase' method followed in TE-1 by three students (see section 3.5.1) was also used here by group two. Both George (mainly) and Niki evaluated the answer and in light of an unreasonable one they used another operation (Sowder, 1989).

In the above extract when George realises that 11 or 1 cannot be a daily wage, he immediately calls Niki to do another operation. In his interaction with Niki ("Niki, check if the multiplication [9.000x5] is <u>right</u>. If the multiplication is <u>right</u> we will cross out the division. We do not use the division."), he uses the word 'right' twice. This word seems to imply that Niki must check if the product of the multiplication can stand as a reasonable answer to the problem, and therefore the multiplication as an acceptable operation for completing the solution.

On the other hand, Niki's responses "Do you mean to divide?" and "That's better." suggest that sometimes she challenged George very suddenly. Niki working in a small group seemed to take the monitoring role in moving the group work forward. Such a behaviour was demonstrated by Niki in a number of occasions. Examples will be presented in later extracts.

However, the students' behaviour suggests that since no-one in group two could see that this was a two step problem and as there was no-one else to challenge George and Niki, this led them to produce a poor performance.

4.4.1.3 Students identifying special roles for themselves

The transcription of tapes revealed that, during collaborative group work, the students in group two did not discuss the situation described in the problem.

They simply chose operations and performed calculations. In addition they appeared in some way to have separate roles. George seemed to be more responsible for selecting the operation/s and Niki seemed to show more responsibility in challenging him and performing the calculations herself. After the problem was given to the groups, Niki's question usually was one of the following:

What shall we do George?
What are we going to do?
What operation shall we use?
Do you mean to add / divide etc.?

After they had performed one or more operations Niki asked George one of the following questions:

What shall we do after, George?
What shall we do now?

Niki was rarely heard in the class, or in the tapes, suggesting an operation. George appeared to be the one who was choosing the operations. In all the sessions they worked together as a group, it seemed that, as far as the choice of operations was concerned, Niki 'blindly' obeyed George. After he had specified an operation Niki's response was: "All right." Then she took the responsibility of carrying out the calculations to complete the solution. She also appeared to guide George through the calculations. George did not force Niki to accept these operations, rather it was she who was looking for George's suggestions.

During the first session of the collaborative group work the diaries entries read:

"George chooses the operation/s for solving the problem. Niki simply listens to him and then starts performing the calculations of the identified operations. Both are annoyed by Argyris' disruptive behaviour."

During the next two sessions the entry reads:

"Niki waits for George to read the problem and say what operation/s they should use. She appears more responsible in calculations than George. She seems to undertake the role of guiding him through the calculations in order to complete the solution. ... George is unwilling to continue with the calculations. It takes some time for Niki to bring him back to work".

At the end of the first week of the actual teaching stage:

"Group two has been working in a similar way throughout the week ... George shows disruptive behaviour. Niki is successful in convincing George to stop making a noise and start working again ... George and Niki refuse to come up to the board."

As soon as George had specified the operations Niki started the calculations. She usually started and finished the calculations before George. She was able to organise the work and lead George towards the end of the solution. When he suggested two or more operations Niki's typical response was: "Okay, let's do that first." After she had finished a calculation she questioned George: "What result did you get?" Although she had not chosen the operations in the problem, and she had not explained them, she appeared more responsible than George in the calculations for the solution.

During the second week of the actual teaching stage in TE-2, when the two groups were joined together, George's behaviour in problem solving changed, while Niki's remained the same. In the larger group, George appeared to have many difficulties in articulating a reason for using a particular operation, but seemed to have an understanding of the situation posed in the problem. Although he made verbal mistakes when he was providing reasons for the required operations, his explanations were valued as acceptable by the other group members, as they were able to understand them. Niki did not demonstrate any change in her behaviour. She continued working in the same way as she did when she was in group two.

4.4.1.4 Writing out the 'thinking procedure' and the explanation of the 'answer'

After the students had solved a problem, they were asked individually to write on their worksheet the 'thinking procedure' of their solution method and the explanation of their 'answer'. That is, how they thought about the problem, and what the answer they found represented. Students had been asked to give reasons for each one of the operations they used. The purpose of this activity was to examine if they understood the problem as well as the operations they used in their solution method.

During this activity, two things were apparent. Firstly the students spent generally more time writing the 'thinking procedure' and the 'answer' than solving the problem. Although they had correctly performed the required operations and they knew the answer (mainly the group one), they encountered many difficulties in spelling out the reason for choosing an operation and/or explaining the answer of the problem. One reason for this behaviour may be that students were not familiar with the activity and so they needed time to accustom themselves to it.

Secondly, it was noticed that all of the students of the first group were consistent with this expectation. In almost all the problems they solved, as their worksheets revealed, the students wrote both the 'thinking procedure' and the explanation of the 'answer'. As the teaching experiment-2 progressed, these students were more adept in meeting these two requirements, and as a consequence they spent less time on them.

In contrast, the students of group two in their 'thinking procedure' were writing only what operations they used without justifying them. In some cases they completely left the 'thinking procedure' and the 'answer' out. One explanation for this is that these students might be choosing the operations by looking at the numbers in the problem, and therefore they could not provide reasons for their choices. Or, they may not have understood the activity and therefore they did not know what they were expected to do.

4.4.2 Students' collaboration during group work

The students collaboration during group work can influence the classroom environment as well as the progress of the group. In this section the students' effort to keep the group work going, their negotiating behaviour and their interdependence will be discussed.

4.4.2.1 Students' effort to keep the group work going

The transcription of tapes and the observations of the collaborative group work, indicated clearly that there was no disruptive behaviour between the students in group one. Two of these three students, Maria and Takis, from the beginning of the actual teaching stage in TE-2, attempted to demonstrate the behaviour they were asked to follow. They proved to be quite successful in following the guidelines. The students of group one appeared to be intrigued by the problem and concentrated on the group work all the time. They discussed the situation in the problem, and their solution method evolved through interaction. By helping and supporting each other they truly developed cooperative skills. Responses such as the following were found in group's one discussion:

T: Okay, we start.

M: Shall I suggest a solution?

T: We should write the 'thinking procedure', and the 'answer' together.

M: Who wants to start?

T: We work as a group. Don't you remember?

M: Say, Stella.

T: Where are you Stella?

M: [Someone from group one asks Stella for something.]
Leave them alone Stella. Let's write our 'thinking procedure'.

One difficulty which appeared in this group involved Stella. Stella was a very shy girl. She showed limited participation in the group's discussion and she refused to reveal her worksheet to the other group members. Maria's and Takis' efforts helped Stella to conform with the group's norms. This case is discussed in a later section of this work.

However, the data collected from group two revealed a different picture. This group basically had two working members, George and Niki. Argyris' presence was rather negative. In fact, Argyris, in two sessions, was asked to work with me, as he had demonstrated extremely disruptive behaviour. In this group, moments of tension were created between George and Niki because of George's unwillingness to continue with the work. But Niki found a way to convince him to resume work each time. Below are cited two such cases from two different problems:

In the earlier stated problem with 'Mr Charlie and his son', George and Niki after three unsuccessful attempts (27.500/9.000, 27.500/5 and 9.000/5.000), performed the multiplication 9.000x5, and their discussion went on as follows:

G: What did you get?

N: 45.000. And you?

G: I got the same, 45.000.

N: What shall we do now? Shall we divide? [9.000/2]

G: [no response]

N: Shall I do the division?

G: I am not going to do anything.

N: Come on George. (*)

G: I am not.

N: Do the division. Come on. Do it. (*)

G: [He starts doing the division 9.000/2.]

In another case both groups worked on the problem:

'A grocer bought 40 cans of cheese at 750 drachmas per kilo. Each can contained 17 kilos of cheese. He sold this cheese at 925 drachmas per kilo. How much was the grocer's profit?'

Group two discussed the problem as follows:

N: What are we going to do?

G: This [750] times this [40] and this [925] times this [17].

N: Multiplication.

G: Yes, 750 times 40 and 925 times 17.

N: Okay. Let's do that [750x40] first.

S: [The students start the first multiplication 750x40. They both finish the first multiplication and they start the second.]

N: Let's go on.

G: [He starts drawing and making a noise.]

N: We are wasting our time George. Come on. (**)

G: [He makes a noise.]

N: Can you be quiet please? (**)

G: [He keeps being noisy, ignoring Niki.]

N: Okay, I will do it at home.

G: Okay Niki. What did you find?

N: 30.000.

Before the analysis of the extracts, I should point out that George is a student who hardly participated in or concentrated during the preliminary observation lessons. He always finds something else to do (i.e. drawing, colouring, playing with his watch etc.). He was never found, during the preliminary sessions, to pay attention to the lesson. And when he finally did, it was because the teacher had noticed him.

During the collaborative group work, although George generally coped well with Niki, there were moments of tension owing to his unwillingness to continue with the work. Niki was able to find a way to bring George back to work.

In the first extract George appeared to be bored or weary after four unsuccessful attempts with the first operation, and was unwilling to continue. Niki did not give way to George's reactions but with two encouraging responses (*) she managed to convince him to start working again. This is consistent with other research findings which have shown that 'there is more facilitative and encouraging interaction among students in cooperative than in competitive or individualistic learning situations' (Johnson and Johnson, 1985, p. 117). Niki's efforts paid off later, when group one and group two combined to form one group. In this situation, as is explained in section 4.4.2.3, it was George who encouraged and helped Niki.

The students' behaviour in the first extract does suggest that between George and Niki there was a lack of communication. When George realised that 45.000 (drachmas) could not stand as a daily wage, he was unwilling to continue with the solution method. It could be argued that George understood the situation and when he saw that something was going wrong he decided to stop. On the other hand, Niki apparently did not realise that something was going wrong in their solution method and therefore pushed George to continue the solution. It becomes evident that although the two students worked together, there was insufficient communication between them and therefore they did not understand each other's actions and behaviour.

In extract two I noticed that George, suddenly abandoned the work and started drawing and making noise. This time he was engaged in more annoying behaviour for his group-mate. Niki attempted twice (**) to encourage George to keep on with the work, but she was unsuccessful. George ignored her each time. This led to Niki's responding in a 'threatening' way: "Okay, I will do it at home." Her response sent George the message that she would no longer continue the work and that the responsibility for doing it would be his. Niki's last response was enough to make George begin to work. It is not clear if that was her intention or not, but it proved to have a positive effect on the group work.

This extract does suggest that group two did not function very well. Niki and George may have misunderstood the activity or they may have found the problems difficult and therefore they could not work out the operations.

4.4.2.2 Leaders with negotiating behaviour

During the actual teaching stage in TE-2, two students, Maria (mainly) and Takis demonstrated leading behaviour. These two students seemed to be the leaders that operated through negotiation. Below is a list with some of Maria's and Takis' responses during the collaborative group work:

S: [After the students had read the problem.]
M: Who wants to start?
M: Who wants to suggest a solution?

S: [The students are assigned to solve a problem in two ways.]
M: Let's do the first way first and then we can talk about the second way.

S: [George and Niki have difficulties with the calculations.]
M: We should wait for George and Niki.

G: [George shows disruptive behaviour.]

M: George, we are not singing now. We are doing maths.

S: [After the students had finished an operation.]
M: Did we all find the same result?

S: [Stella had correctly suggested and justified the required operations for solving a problem.]
M: Exactly. 30 times 17, 40 times 8, and then to the sum of these two we will add the 120. That is three operations.]

S: [After a group member had suggested an operation without explaining why.]
T: Yes, but can you explain why?

G: [George has difficulties with the second way of solving the problem.]
T: Don't worry George. We'll do it together.

S: [The students have to perform the straightforward multiplications 100x80 and 100x45.]
T: Let each of us perform the multiplications alone and then we'll check the results. How is that?

G: [When George specifies an operation, but avoids mentioning the numbers (i.e. this times this, this plus this, etc.) , he causes confusion to the other group members who then ask.]
T: Can you tell us the numbers please?

Both students Takis and Maria had the ability to stimulate the negotiating behaviour in other members of the group. They were seen to initiate solutions and suggestions, raise questions and challenge statements and assertions in ways that enabled the discussion to move forward. They were assessing the progress of the group and they were seeking to gain the agreement on discussions that were taking place.

Particularly for Maria one of the diary extracts read:

"She takes it upon herself to be the leader of the group. She is a popular girl in class and (one of) the highest attainers. The group accepts this."

On the other hand, the low-attainers, Niki and George, could be successful leaders in the large group if they could solve the problems. Their behaviour in group two suggested that both or either of them could undertake the role of leader.

4.4.2.3 Students' interdependence

When George and Niki were in a separate group, it was Niki who was encouraging George to continue the group work and she also supported him in the calculations. However, when they moved into one group, it was noticed, that the behaviour of these two students was reversed. It now was George who encouraged and helped Niki. George seemed to be able to follow Maria, Takis and Stella who were competent in calculations, but Niki appeared to have difficulties and sometimes was staying behind.

It could be argued that George seemed better in calculations for two reasons. First he may have thought that he was placed in a wrong group, with Argyris and Niki, and that he did not belong there. Later, however, when he was in a better group, he knew what to do. Second, George was sitting next to Takis who was comfortable with the calculations, and received help from him.

The two students' change of behaviour is illustrated by their responses to the calculations on the following solution method. The group had been assigned to solve a problem with two ways. The students had finished the first way that required four operations (1.450x35, 1.500x35, 2.780x35, 50.750+52.500+97.300) and they had started working on the second way. Niki had not finished the first way, as she had difficulties with the multiplication

2.780x35 and had stayed behind the rest of the group. The interaction between George and Niki went as follows:

N: Why do I keep doing it wrong?

G: Do it again Niki. Slowly.

N: Somebody do it for me. [She appears reluctant to repeat the calculations.]

G: Come on Niki.

N: Let me just check. [She starts checking the multiplication.]

M: You don't have to hurry Niki. Argyris is still behind.

N: I am still confused.

G: [George who has finished the multiplication gives his worksheet to Niki.]

N: [She appears to be relieved.]

Niki finds difficulties with the multiplication 2.780x35 and complains about it. Her responses "Why do I keep doing it wrong?" and "Somebody do it for me." are requests for explicit help. George's initial effort to encourage Niki brings no result, as she appears reluctant to follow his suggestion. George realising that, responds with a more stimulating prompt, "Come on Niki." This time he is successful as Niki starts checking the multiplication again. Maria then encourages Niki. She tells her not to rush as Argyris is still behind, although Maria knows very well that Argyris simply copies Takis' work. Niki's checking of the multiplication does not seem to help her to find her error and she says that she is confused. George then takes his worksheet and shows it to Niki. She immediately spots the mistake and is relieved. The interdependence (Johnson and Johnson 1985, 1992) between the two students tended to make George feel responsible for Niki's performance. On a number of other occasions, when students were checking the results of the operations it was noticed that George and Niki checked each other's.

It could be argued that the positive interdependence developed between Niki and George, when they worked in a separate group, also pursued them in the larger group with all the students. George seemed to feel responsible for Niki's productivity and obliged to support and assist her (Johnson and Johnson, 1992). But now their behaviours were reversed. George now assists Niki with the work. Niki's earlier efforts to convince George to keep going with the work seemed to have succeeded. Although other group members supported Niki too, it was George who was the one who did most of the encouraging.

4.4.3 The group's influence on students' behaviour

Working collaboratively in a mixed ability group seemed to have implications for students' attitudes and learning behaviour. In this section will be discussed how a student's self-confidence was built up, how communicating understanding led to strengthening that understanding, how students' behaviour improved, and how a special case was dealt with.

4.4.3.1 Building up students' self-confidence

As stated in an earlier section, one difficulty that had arisen in group one during collaborative group work, concerned the case of Stella. She was a very shy girl who showed limited participation in the group's discussion and refused to reveal her work to the other group members. An early tape revealed:

The students of group one had just performed the division 27.700/5 and Maria asked Stella for her answer:

M: [to Stella] : What did you get?
S: [She refuses to show her worksheet. She hugs it close to her.]
5.500
M: We are one team Stella.
S: I know.
T: We do not hide our work.
S: I'm not hiding it.

M: What are you doing then? What's this? (*)

[She refers to Stella's behaviour.]

S: [no response.]

T: We should work together as a team.

The above extract is from the second session of teaching experiment-2. It is also the second time that Maria and Takis encounter the same reaction from Stella. A similar interaction had taken place at the initial session between Stella and Maria. This time, Takis is also involved. It is noticed that Stella answers Maria's question but she refuses to show her worksheet. Then both Takis and Maria make an effort to convince her that they all constitute a team and that they should work collaboratively. Obviously Maria cannot accept Stella's response, "I'm not hiding it", and pushes the matter even further with the double question she addresses: "What are you doing then? What's this?" Stella then replies with silence.

In the initial session of collaborative group work, diary extracts also revealed that Stella was having some difficulties with the collaborative environment. The diary's entries read:

"Stella listens to the group's discussion but she rarely participates. She participates only when she is asked. She performs most of the calculations in the operations correctly, but refuses to show her work (the worksheet) to the other group members, despite being asked. ... There is a confrontation between Maria and Stella about Stella's attitude. Stella copies Maria's 'thinking procedure'."

Later extracts reveal:

"Stella who most of the times refuses to show her work has begun to be more communicative. She does not keep her work covered but she shows it to her group-mates, yet at the same time protects it. She holds the worksheet with both hands. She appears to participate more in the group's discussion."

The following week, when both groups (one and two) are combined:

"Stella is working well with the other group members. She discusses with her group-mates, ways of approaching the solution to the problem at hand. She appears to be happy. She does not hide her worksheet and her work is available to all group members. She does not hold the worksheet either."

Working collaboratively in a mixed ability group (of narrow range) helped Stella to build up her self-confidence. Her group-mates, Maria and Takis, helped Stella in a number of ways during group discussion. The transcription of tapes revealed that Takis and Maria in the initial period of the actual teaching stage in TE-2, many times used responses such as the following:

S: [The students read the problem.]
T: What do you think Stella?

S: [Stella wants to suggest a solution.]
M: Okay. Say it, Stella.

S: [Maria and Takis finished the calculations but Stella is still behind.]
T: Wait for Stella.

S: [After the students had performed all the calculations.]
T: What answer did you get Stella?

S: [She refuses to show her worksheet.]
M: We should work as a team Stella.

S: [Stella suggested an appropriate solution method.]

M: Exactly, Stella.

Maria and Takis avoided criticising Stella's work and attitude (except the first two times) during group work. The reinforcement and encouragement (Johnson and Johnson, 1985) they used, in their prompts towards Stella, helped her to build up her self-confidence, to start slowly open up and gradually to change her attitude. Both students' behaviour seemed to have a positive influence on Stella's motivation towards collaborative group work. Stella, being in such a supportive environment, after 4 sessions, began to participate more in the group's discussions and to make her work available to the other group members.

On the other hand, Stella being a student who always concentrated during the collaborative work could see that she could do the work and thus contribute to the group. This is another reason that may have resulted in building up Stella's self-confidence and become active in group work.

Overall, it seems that the building up of Stella's confidence may have come from two factors: acceptance by the group and her ability to do the work. Takis and Maria did not reject her because she was behaving in a particular way, but rather, accepted this and helped her to conform to the group's norms. At the same time Stella herself could see that she was able to do the group work, thus contributing to the common goal. These two reasons may have made a great contribution in helping Stella to build up her self-confidence, and could have led therefore to a change in her attitude and behaviour towards group work in problem solving.

4.4.3.2 Communicating one's understanding leads to strengthening that understanding

Opportunities for learning mathematics, when the children worked in groups, arose when one child attempted to assist another who had misunderstood the situation within the problem or the outcome of an operation. The child who assumed the role of tutor had to extend his/her own understanding of the situation, so as to give an explanation that would make sense to his/her group-mate. Previous research into small group work has indicated that giving help and explanations is positively related to students' achievement (Webb, 1991).

Below two such cases are discussed. One when all students worked as a group (first extract) and another when they worked in two groups (second extract). The students had been assigned to work on the following problem:

'The owner of a coffee-shop bought 5 dozen glasses and paid 12.000 drachmas. How much do 4 of these glasses cost?'

The discussion of this problem went as follows:

S: [The students read the problem.]
M: What do you think?
G: We should divide this [12.000] by this [5] and then ... [pause]
M: Times 4.
T: Yes, but can you explain why?
G: 12.000 by 5 to find out how much each glass costs.
S: And then times 4 to find out the 4 glasses.
T: Do we all agree?
N: Yes Takis.
T: Okay, let's do it then.
S: [The students start to perform the calculations.]
M: Just a minute ... [pause] I think it's not quite right.
T: What's wrong?
M: I think we should divide by 60. Don't we?
N: By 5 or by 60?
T: Why by 60?
M: Because it's says 5 dozen.

T: Oh! Yes. I've got it now.

S: Yes. Yes. You are right. 5 dozen is 60.

N: Okay. Let's do the division. 12.000 by 60.

G: I don't get it.

M: Look George. It says that he bought 5 dozen glasses not 5 glasses.

G: Okay.

T: But you said to divide by 5.

G: Yes.

M: George, it says here 5 dozen ... 5 dozen are 60 glasses because 5 times 12 is 60. He paid 12.000 drachmas for 60 glasses not for 5.

S: That would be stupid. To pay 12.000 for 5 glasses.

G: Okay. I've got it now. 5 dozen. I did not notice 'dozen'.

...

T: I did not expect such a small problem to take us so long.

At the beginning of the discussion, all students seemed to agree with George that a division 12.000 by 5 should be one of the two required operations for solving the problem. However, when students started to perform the division, Maria intervened in a challenging way by saying that they should divide by 60, because there are 5 dozen. Her intervention made sense to Takis and Stella, who both immediately noticed the mistake and agreed with Maria. This however, did not happen with George who had obviously misunderstood the information '5 dozen' and had read it as simply 5.

Maria and Stella had mentioned in their responses "... 5 dozen" and "... 5 dozen is 60" respectively, but George could not see where the 60 came from. Maria assumed the role of tutor attempted to explain to George that the coffee-owner bought 5 dozen glasses and not 5 glasses. Despite her explanation and Takis clarification, George persisted on 5. Then Maria, in an effort to communicate her understanding of "5 dozen" to George, extended her own conceptualisation of the problem (Yackel et al., 1991) and came up with a more elaborate answer. First she explained to George in a mathematical way where the 60 (5x12=60) came from, and second she put the situation in a real context by saying that the coffee-owner paid 12.000 drachmas for 60 glasses and not for 5 glasses.

In another case, when the students worked in two groups, they had to solve the problem:

'Koula has saved 13.800 drachmas and her brother 960 drachmas. How many drachmas should Koula give to her brother so that both have the same amount?'

The interaction in group one concerning this problem was:

M: Let's start.
T: We will subtract.
M: Yes. I think we should do a subtraction to find out how much Koula should give to her brother so that they both have the same amount.
T: We should subtract to find out how many extra drachmas Koula has and then ... [He finishes his suggestion here.]
S: [The students start perform the subtraction 1.380-960.]
M: What did you get Stella?
S: 420
T: I have the same.
M: Let's write the 'thinking procedure' and the 'answer' now.
S: [The students follow Maria's suggestion but Takis suddenly stops and comments] :
T : It's wrong.
M: Why?
T: What if she gives all of them [420] to him?
S: That's what she should do.
T : We forgot something.
S: [pause]
M: Division.

T: Did you understand it?

M: Yes.

S: Not quite.

T: 420 are not the money that Koula should give [to her brother] but how much extra she got. She should give half of them.

S: Should we divide?

T: Exactly. Take the extra money, the 420 drachmas, split it in half and give it to each other. Thus divide 420 by 2.

S: I've got it. She should give half of the extra money [to her brother.]

M: Good thinking Takis.

S: [The students start performing the division 420/2.]

In the above extract Maria suggested a subtraction in order to find out how much Koula should give to her brother. Takis pointed out that they would find out how much extra money Koula has, but he finished his suggestion at this point as he probably could not see how to proceed further towards the solution. However, his phrase "and then ..." indicates that Takis was thinking of another operation for solving the problem but was not sure of it. Although Takis expressed a different reason for subtracting, it seemed that Maria did not grasp his point. This became evident, after the subtraction, when she called her group-mates to write the 'thinking procedure' and the 'answer' to the problem.

Takis, realising that 420 is not the amount that Koula should give to her brother, responds with an affirmative impersonal statement: "It's wrong." He does not explain why but he challenges the other two members. The challenge together with the offered hint "We forgot something.", helped Maria to realise that a division was required to complete the solution. When Takis asks the rest if they understood it, Stella responds negatively.

Takis then adopting the role of tutor, explains to Stella that the 420 drachmas do not represent the amount that Koula should give to her brother but how much extra money she has, and that she should give him half of this amount.

Despite Takis' explanation, Stella responds with a question. She is still uncertain about the division. Finally, Takis specifies three specific actions (i.e. take, split, give) to Stella in order to justify the division. Takis draws on his own understanding of division in order to develop an explanation that will make sense to her. It was observed that the high-attainers, Maria and Takis, undertook the role of the tutor in 8 problems, and in 5 of them they came up with more elaborate explanations.

In this problem, Takis and in the previous problem, Maria, had to construct 'a more elaborate conceptualisation' of the situation in order to communicate it to their group members. In their effort to do this, their own understanding tended to be strengthened. It was noticed that learning opportunities like these arose during collaborative group work.

4.4.3.3 Students improving their behaviour

Collaborative work in one group of a wide range of mixed abilities seemed to be beneficial, in different aspects, to four of the six students and particularly for George.

Early on the actual teaching stage, Maria and Takis would explain and justify their choices of operations. As discussed in the previous section, a result of the effort to communicate their understanding of a solution was that the understanding seemed generally strengthened. This behaviour was evident in both ways of working, that is, when Maria, Takis and Stella worked as a separate group (group one) but also when they joined group two.

Stella who was a very shy girl with a limited participation in group discussion, showed significant changes in her behaviour during the second week of teaching experiment-2 (all students in one group). As was explained earlier Takis and Maria may have played a crucial role and thus contributed strongly to this change. Their support, appeared to have helped Stella build up her self-confidence, to start reassure herself and finally to participate in the group as an equal member. Her lack of explanations during the pre-interview, seemed to be due to the fact that she was unaccustomed to justify her choice of operations in the class.

In addition to the above changes of students attitudes and learning behaviour, collaborative work in one group (of mixed abilities) proved to be particularly beneficial for George. When he was in the same group with Niki and Argyris, he would identify the operations and look for the answer to the problem, by examining the results of different operations (with different combinations of numbers). However, when all the students worked as one group, George demonstrated a change in his behaviour. But this was not the case with Niki. The following tape extract illustrates this point:

The students had been assigned to solve the following problem in two ways:

'The book-seller sold in a day:
35 literacy books for 1.450 drachmas each.
35 history books for 1.500 drachmas each.
35 scientific books for 2.780 drachmas each.
How much did he collect ?' (MNER, 1993a, p. 117).

S: First we will do a multiplication 35 times 1.450 to find out how much he collected from the literacy books.
[Stella starts the multiplication.]
M: Stella, let's talk about the problem first and then we shall do it all together.
S: All right.
T: After we will do the same thing for the history and the scientific books.
M: Okay. First we do three multiplications. 35 times 1.450, 35 times 1.500, 35 times 2.780.
G: And then we add them together to find out how much he collected totally.
M: Let's first do the first way and then we can talk about the second.

...

S: [The students finished the first way and they were ready start talking about the second.]

T: Let's try to solve the problem in the second way.

N: We did four ways. [Niki obviously counted the operations: three multiplications and one addition.]

M: Niki, this was the first way of solving the problem.

N: Oh!

When students discussed the solution of the first way, George participated actively. He suggested an operation and provided a reason for choosing it. By using the word "totally" he strengthened his explanation. George has now understood what is required. Such a behaviour was previously, unusual, for him. The environment of the new group, enriched by more able students than George, challenging and supporting each other, appeared to exert an influence on his behaviour. Contrary to George, Niki seemed to work as before. She performs operations without understanding why. When Takis prompts his group-mates to think of a second solution method, Niki gives an incorrect answer. She says that they had used four different ways. She seemed to have confused the number of operations with the number of solution methods. Niki's behaviour during the actual teaching stage in TE-2, explains her intuition based explanations during the pre-interview procedure.

In another case the students were asked to solve the following problem in two ways:

'The father earns from his work 8.150 drachmas per day and the mother 7.990 drachmas. How much do they get both per month? (25 working days).' (MNER, 1993a, p. 117).

G: I think I know what we should do.

M: Okay George. Tell us.

G: We do a multiplication 8.150 times 25 to find out how much the father earns per month, and then 7.990 times 25 to find out how much the mother earns per month.

S: Exactly. And then we add them to see how much they both get.

M: Per month.

S: That what I meant.

T: All right. 8.150 times 25.

G: We know.

T: And then 7.990 times 25.

G: Yes. But do you understand why?
Because there are 25 working days.

The students continued and solved the problem in both ways. The first way required two multiplications and one addition while the second way required one addition and one multiplication. When the class discussion on this problem started, Niki was asked to present the first way of solving the problem. What follows is the interaction that took place:

I : Niki, can you explain to us the first way of solving this problem?

N: [She counts the total number of operations in her worksheet.]
We did three multiplications and two additions.

M: Why three multiplications?

N: [She counts: 1, 2, 3.]

S: The third multiplication belongs to the second way. Now we are discussing the first way.

N: Oh! We did two multiplications and one addition.

I : Why did you do these operations?

N: [no response]

I : What was the first multiplication?

N: 25 times 8.150.

I : Why did you think of multiplying 25 by 8.150?

N: Can you ask somebody else, sir?

I : Who wants to explain to me why you multiplied 25 times 8.150.

G: Can I tell you sir?

I : Yes, George.

G: To find out how much the father gets in 25 days from his work. We know how much he gets per day but we do not know how much he gets in 25 days. That's why we did this multiplication.

George's learning behaviour during problem solving appeared have improved from before, when he was in a separate group. He stopped choosing operations by looking at the numbers of the problem, as nobody else in the group was following this practice. George seemed to be conforming to the group's expectations, where everyone who suggested an operation had to explain it. This benefited George as now he was constructing meanings for the operations he used. On the other hand Niki's behaviour remained the same. The new group environment did not seem to have an influence on her, although some changes were expected.

4.4.3.4 A special case

In the first session of teaching experiment-2, all group members appeared involved except Argyris who was behaving in a disruptive manner. Since Argyris could not work with Niki and George he started to interfere in their working together. In the second session the diary read:

"Argyris head-butted George. George is very upset ... Argyris starts whistling and Niki looks very annoyed. Argyris is removed from the group ... He is being too disruptive ... We agreed that he would work with the researcher for the time being."

Argyris could not work with his group-mates. When he was removed from the group he settled down immediately. It was noticed as time passed and as the group members appeared content and involved, that Argyris was looking across at his group. However the second session was completed with Argyris working apart from his group.

I faced many dilemmas concerning the case of Argyris. Although Niki and George did not make any complaints about Argyris' behaviour, they looked very annoyed. His lack of basic knowledge may have been one reason for the difficulties he had faced with the group work. At the beginning of collaborative group work it did not seem appropriate to leave him out of a group. When, however, he started to show disruptive behaviour, I removed him (from his group) in order to let the other two members work without distractions.

At the beginning of the fourth session Argyris went voluntarily and rejoined with his group. Immediately I asked him to work separately, as in the second session. Then he burst into tears and he begged to stay with the group promising that he would be quiet. I gave Argyris another chance and allowed him to work with his group. During this session it was noticed that his behaviour had changed. He appeared to realise that to work with the group, his behaviour had to be acceptable to them and to the teacher: "He worked well during the whole session listening to the group and showing acceptable behaviour." Afterwards, Argyris continued to work with the group, until the end of the actual teaching stage, without causing any further disruption. Overall his behaviour appeared to have improved beyond expectations.

Argyris demonstrated a different behaviour than Vasilis in teaching experiment-1. Argyris after he was removed from his group, he seemed to be adversely affected by this and therefore he voluntarily rejoined his group. Consequently, he conformed his behaviour to that of the group's expectation. On the other hand Vasilis did not seem to be emotionally affected by removing him from his original group and he continued to show unacceptable behaviour.

4.5 Changes in students' attainment and attitude

4.5.1 Changes in students' attainment

The post-test (Figure 4.6) and the post-interview took place four weeks after the actual teaching stage. The post-test administered to the students was identical to the pre-test. Again, as in the pre-interview, the post-interview procedure started immediately after the students had completed the test and were then asked to explain and justify their solution method. The objective of the post-test and post-interview was to examine if students had treated the five word problems in the test differently from the way they had before the actual teaching stage. The results of the pre-test and the post-test, and the number of correct solutions to each problem and each test, are shown in the Figure 4.7 and in the Table 4.1 respectively.

The results of the pre-test and the post-test (given in detail in Appendix 6) suggest that most students performed better at the post-test than the pre-test. The students who failed to demonstrate any improvement were the low attainers (Niki and Argyris). Furthermore, during the post-interview it was noticed that the students, who had improved their performance at the post-test, developed competence in explaining and justifying their solution method. This behaviour was also found in Maria's case.

In the following paragraphs, each student's performance and reasoning on the problems in the post-test is discussed with reference to the interviews.

Maria

When Maria was asked about her solution method in the first problem the following interaction took place:

I : Maria, what was your reasoning in the first problem?

M: I thought I would do a division 312/12, because we know how many

Figure 4.6 Test given to students before and after the actual teaching

Pre-test / Post-test		
Class: D		
Name:...		
Problem 1		
312 photographs must be placed in an album.	312x12	12+12
On each page of the album should be placed	12/312	312-12
12 photographs.	12+312	12x26
	12-312	312/12
How do you work out how many pages are		
needed?		
Problem 2		
Everyday Pericles' father goes to work and	27x22	22-27
comes back with his car and covers 27 km.	22+27	27-22
	594/27	22/27
How many km does he cover in 22 days?	22x27	27+27
Problem 3		
A truck driver has to drive 520 km to get from	260x2	520+260
Athens to Salonica. After driving 260 km he stops	260/520	520-260
for lunch.	520x260	520/260
	260-520	260+520
How do you work out how far he still has to drive?		
Problem 4		
37 men and 23 women are working in a factory. How many hours of work do		
they complete together in a day (eight hours)?		
Problem 5		
The mother bought 10 cans of milk at 100 drachmas per can, 4 kilos of sugar		
at 150 drachmas per kilo and one kilo of cheese. She paid 3.600 drachmas		
for all these.		
How much did she pay for the cheese?		

Figure 4.7 Students' performance on the pre-test and post-test

	Maria		Takis		Stella		George		Niki		Argyris	
	Pre	Po	Pre	Po	Pre	Po	Pre	Po	Pre	Po	Pre	Po
Problem 1	V	V	X	X	X	V	X	X	V	X	X	X
Problem 2	V	V	V	V	X***	V	X	V	V	X	X	
Problem 3	V	V	V	V	X	X	X	V	X	X	X	X
Problem 4	V	V	V*	V	V*	V	V**	V	X	X	X	X
Problem 5	V	V	X	V	V	V	X	V	X	X	X	X

* They did not identify the last operation (addition).

** After the two multiplications he interpreted the results in an odd way without combining them.

*** When she started explaining her answer, she immediately realised her mistake and proceeded to give an explanation for the correct mathematical expression.

Table 4.1 Number of correct solutions to each problem and each test

	Number of correct solutions	
	Pre-test	**Post-test**
Problem 1	2	2
Problem 2	5	3^
Problem 3	2	3
Problem 4	1^	4
Problem 5	2	4

^ With the distinction made in the above table.

photographs we have, we know how many photographs can be placed on each page of the album and we want to find out how many pages are needed. I chose division because I know how many photographs there are altogether and I also know how many can be placed on each page.

I : What will you get from this division?

M: How many pages are needed to place these 312 photographs.

Maria's explanation suggests a spatial image of the problem. Conceptualising the structure of the problem (Krutetskii, 1976), Maria describes her visual image (of the problem) with great clarity. She does not refer to the logic but rather to the picture she has found. Every time Maria was explaining a solution method, she used her own personal way to present both the given and unknown information of the problem. By organising the information appropriately and by using intonation she indirectly outlined the required operation.

In the second problem, Maria ringed the correct mathematical expressions, 27x22 and 22x27, as she had done in the pre-test, and she explained them appropriately.

Maria's explanation for the third problem was characteristic, and it was revealed in the following interaction:

I : Maria what was your reasoning in this problem?

M: We know how many km he <u>should</u> cover.
We know how many km he <u>has</u> covered and we want to <u>find out</u>
how many km he <u>will</u> cover.

I : How are you going to find it then?

M: We will subtract. From the 520 km he should cover, we will subtract those that he has already covered, that is 260. And then what we will get, it will be how many more km he will have to cover.

It was noticed that before identifying the operation Maria, reformulates the problem using the textual information in the problem. By doing this reformulation, she indirectly outlines the required operation. She then responds to the (researcher's) prompt, by specifying the necessary operation and incorporating the numerical information of the problem to her explanation. The transcription of tapes reveal that Maria engaged in the behaviour discussed above on a number of occasions during collaborative group work. She particularly tended to do this when she was attempting to explain things to her group members. As a result of this behaviour, every time she was assuming the role of the tutor, her understanding appeared reinforced. Maria's correct explanation of the fourth problem was the following:

M: First, I had to find out how many people there were all together. So I added 37 which was the men and 23 which was the women and I found 60. Then I multiplied it by 8, in order to find out how many hours they work together in a day, if everyone works 8 hours. And I found out 480 hours.

Maria gave also a correct explanation for the fifth problem. However, I should point out here that Maria's thinking appeared improved in comparison to the pre-test. In the post-test she explained and justified her solution method spontaneously, while in the pre-test I had to ask her a number of questions before she gave me the required explanation.

Maria's mathematical thinking appeared in many cases to be different from her classmates. She thought of solutions and suggested approaches that the other students could not think of. Maria's unique way of reasoning was probably one of the reasons that she was an accepted leader in the group (and class).

Takis

In the first problem, Takis ringed the wrong mathematical expression 312x12, as he had done in the pre-test. During the interview, he explained: "The first problem was the only problem in which I did not think the operation through beforehand, but rather I looked at the expressions and I ringed the one I thought was correct."

Takis, when questioned about his solution method in the second problem, explained: "27 times 22 or 22 times 27. I know how many km he covers in one day and I want to find how many km he covers in 22 days. If we multiply 27 times 22, we will find out how many km he covers in 22 days. If we add 22+27 what will we get ? Days and Km ? Nothing relative to the problem."

Takis explained and justified his choice of operation. Moreover, he continued the investigation to the expression 22+27, which was placed exactly below 27x22, and showed why it was not an acceptable solution to the problem.

In the third problem, Takis ringed the correct mathematical expression 520-260 and he explained it appropriately.

Takis correct explanation for the fourth problem was the following:

T: In this problem I had to identify the operations because they were not given. ... I multiplied 37 by 8 to find out how many hours the men work in a day. Then I multiplied 23 by 8 to find out how many hours all the women work in one day. At the end I added their hours and I found out how many hours they complete together, if everyone works eight hours per day.

Takis explained his method for working out the fifth problem as follows:

T: At the beginning I did two multiplications to find out how much she paid for the 10 cans of milk and how much she paid for the 4 kilos of sugar. Then I added these two to find out how much she paid for both together. And then I subtracted it [the sum] from how much she paid totally, and I found out how much she paid for the one kilo of cheese. 2.000 drachmas.

Takis identified the necessary operations and he did not repeat the mistake he had made in the pre-test. In his explanation, it was noticed that he used mainly textual, and limited numerical information.

Takis, as it will be shown also at the post-questionnaire, was a student who wanted to finish his work as quickly as possible. In two cases in the pre-interview (regarding the third problem and the fourth problem) he had commented: "I didn't notice that, I was in a hurry." (in the third problem), and "Oh! Oh! I forgot to do an addition ... I read the problem quickly." (in the fourth problem). Collaborative group work may have helped Takis to concentrate more on his work and thus to improve his performance. Takis appeared to have an understanding of the operations and how to apply them to the various problems. However in the pre-test he tended to make errors, because he may have wanted to finish before his classmates and therefore was not giving the required attention to the problems. It may also have been because of his attitude to testing. In the post-test, however, he performed very well and at the followed interview (post-interview) demonstrated an improved behaviour. In the pre-test he solved two problems and half of another out of five. In the post-test he solved four.

Stella

Stella in the first problem ringed the mathematical expressions 312/12 and 12/312. When she was interviewed about this problem the discussion was as follows:

I : Stella, can you explain to me your reasoning in the first problem ?

S: I thought I must divide.

I : What made you think of dividing ?

S: In order to find out how many pages are needed for the 312 photographs. On each page, there can be 12 photographs placed. If we divide, we will know how many pages are needed.

I : What will you divide?

S: 312/12 or 12/312. [She points on the test where both expressions are ringed.]

In the post-interview Stella identified the required operation and justified it. At the pre-interview Stella had difficulties providing justifications for the operations and every time she responded looked for approval. At the post-interview, she exhibited different behaviour. She answered the questions directly and did not seek approval. It could be argued that Stella's collaborative group work experiences, where she had to provide reasons for her choices of operations, may have had an effect on her behaviour at the post-interview.

In the second problem, Stella had ringed the correct expression 27x22 in the pre-test, but in the post-test she selected a different and incorrect choice of operation, i.e. 22/27. When questioned about the second problem the following interaction took place:

I : Stella can you explain to me your reasoning in the second problem?
S: I did a division.
I : Why did you think to divide?
S: I divided 22 by 27. But it's wrong. 27 doesn't go to 22. We should multiply 27 times 22.
I : Can you explain why?
S: To find out how many km he covers in 22 days. We know how many km he covers in one day and we want to find out how many km he covers in 22 days.

While Stella was explaining her solution method, she noticed her mistake. After she started the explanation, she immediately realised her mistake, and proceeded to offer an alternative solution. Stella did not give an explanation for the incorrect mathematical expression 22/27, but did give an explanation for the correct one.

In the third problem, Stella was unsuccessful. During the interview although she explained the incorrect expression, she did not manage to point out the correct one.

Stella's correct explanations for the last two problems (fourth and fifth) were the following:

S: I multiplied 37 by 8 to find out how many hours the men work in a day.

After, I multiplied 23 by 8 to find out how many hours the women work in a day. Then I added the men's hours and the women's hours to find out how many hours they work together in a day.

S: First I multiplied 10 by 100 to find out how much she paid for the 10 cans of milk. After, I did another multiplication 4 by 150 to find out how much she paid for the sugar. Then I did an addition to find out how much she paid for both the milk and the sugar, and I got 1.600. Then, I subtracted it from 3.600 and I found out how much the cheese cost.

In the post-interview Stella did not encounter any difficulty in explaining her choice of operations and probably being aware of the type of explanation wanted, changed her behaviour and appeared to have conformed to the required type of explanation.

George

In the first problem, George ringed the incorrect mathematical expression 312x12, as he had done in the pre-test.

In the second problem, George, had ringed the correct expression 27x22 in the pre-test, but in the post-test he selected a different, incorrect choice of operation, i.e. 22+27. When questioned about the second problem the following interaction took place:

I : George, what was your reasoning in the second problem?

G: I added 22 plus 27 and 27 plus 22. The answer is the same.

I : Why did you think to add them George ?

G: He covers 27 km in one day and we want to find out how many km he will cover in 22 days. I added them up.

I : What did you add?

G: 27 plus 22. [pause] Km and days. Oh! Oh! You cannot do that.

I : Why?

G: We cannot add km and days.

I : Can you explain that?

G: They have to be the same. We only add same things.

George's responses indicate that he is aware of the addition's commutative law, but he may have misunderstood the situation described in the problem. After prompting, he managed to point out his mistake and also to provide a reason for his error. However, he did not indicate the correct operation for the problem.

In the third problem, George was one of the three students who ringed the correct mathematical expression 520-260. When questioned about this problem he explained: "We will do a subtraction 260 from 520. ... It says that the truck driver has to drive 520 km, but when he stops for lunch he has driven 260 km. So we have to subtract 260 from 520 in order to find out how many more km he will have to drive".

George's explanation indicates that he had understood the given information. He chose an operation and he provided a reason for this. Probably being aware of the fact that the commutative law does not apply to subtraction, he ringed only the correct mathematical expression. It is worth mentioning that at the pre-test he ringed the expressions 520+260, 260+520. He might then have misunderstood the problem's information, as he had explained that the driver covers 520 km from Athens to Salonica and 260 extra km to go to the place where he stops for lunch.

George's attitude towards collaborative group work, where they had to read the problem carefully, to specify the operations and to justify them, seemed to have an influence on his performance. In the pre-test he had successfully solved one problem and half of another and in the post-test solved three. In addition, it was observed that, during the post-test in class he appeared more concentrated in his work.

George's correct explanations for the fourth and the fifth problem were the following:

G: I thought I must multiply 37 by 8 to find out how many hours all the men work in one day. And 23 by 8 to find out how many hours the women work in one day. Then I added these and I found out how many hours the men and the women work together in one day.

G: I did 10 times 100 to find out how much she paid for the milk. I did 4 times 150 to find out how much she paid for the sugar. I added them up and I found out that both of them cost 1.600 drachmas. Then I subtracted it from 3.600 and I found out how much the cheese cost.

George identified the necessary operations and he did not repeat the mistake he had made in the pre-test.

George had concentration problems. During the preliminary sessions intervention was required by the teacher to bring his attention back to the lesson. In the first week of the actual teaching stage, he worked with Niki and Argyris and demonstrated disruptive behaviour. Niki's effort helped to bring him back to the group work. George chose operations without providing explanations. Furthermore, he looked for the answer to the problem by evaluating the reasonableness of the results in different operations.

When George was in the large group (all students together) he tended to behave in a different way. Following the group's norms, he appeared to be concentrating on the work and changed his way of discussing a solution method. Each time he suggested an operation, he provided a reason. The change of George's behaviour may have had an influence on his performance on the post-test.

Niki

In the first problem, Niki ringed the expression 312x12. When she was questioned about her choice of operation she explained: "Since you had asked us to do it before, I remembered that I had ringed the first operation 312x12. [Pause] I remembered what I had ringed the first time and now I did the same."

When questioned again regarding her answer choice 312x12, she replied in a similar way. Providing this explanation she avoided justifying mathematically the operation she had chosen. However, her explanation was not consistent with the operation she had chosen in the pre-test. In this Niki ringed the correct mathematical expression 312/12, but she attributed her answer to intuition.

In the second problem, Niki was one of the three students who ringed the correct mathematical expression (27x22), but she provided an explanation which was not based on a mathematical argument. During the post-interview, when she was asked to justify the operation 27x22, she responded as follows: "I ringed 27 times 22. I had the feeling that led me to believe that it was the correct one ... [pause] Something told me, that this expression, was the correct one."

Niki identified and ringed the correct mathematical operation but she could not justify her answer. Her explanation was based on intuition. During the post-interview she did not seem to have any understanding of the operation she had ringed. At the pre-test she had chosen the correct mathematical expression but she had given another unexpected explanation (i.e. "My mother has a book with problems. Sometimes we solve problems like this. That's why.")

In the third problem Niki, Stella and Argyris ringed an incorrect mathematical expression. When Niki was asked to explain her solution method the following discussion took place:

I : Niki, can you explain to me your reasoning in the third problem?

N: I ringed the division [520/260], but now I think that I should have ringed the multiplication.

I : Why did you ring division?

N: I don't know. [This is the first time that Niki uses this response.]

I : You also mentioned multiplication. Why do you think you should ring multiplication?

N: Something inside me tells me to do this. I don't know.

I : So, what do you think you should use in this problem ?

N: Multiplication.

I : Why ?

N: [no response]

I : Niki, what route does the driver has to follow?

N: From Athens to Salonica.

I : How many km does he have to cover?

N: 520.

I : Has he covered any km so far?

N: Yes, 260.

I : What does he do after?

N: He stops for lunch.

I : What does the problem ask you to find?

N: How many more km he has to cover. Division?

I : Why?

N: Subtraction?

I : Why?

N: Because we have done this in class ... [pause] I remember now, something like this ... [pause] but not exactly the same ... [pause] in class with our teacher. And I remember that we used subtraction.

It was the first time (since the beginning of the actual teaching stage in TE-2) that Niki responded with the phrase "I don't know." She had never been heard to use this phrase so far. When Niki is asked about her choices of operations she replies with silence or she offers explanations which are not related to mathematical reasoning. Each time she was forced to justify an operation she shifted the emphasis to irrelevant explanations. The effort made through the sequence of prompts to lead Niki to the required operation, was rather unsuccessful. Although she could cope with the computations, she gave the impression that she could not apply the operations to the solution of problems.

Niki in solving the fourth problem and the fifth problem was unsuccessful. The following interview extract explains her behaviour in the fourth problem:

I : Niki can you explain to me how you approached the solution to the fourth problem?

N: I thought .. [pause] I read the problem so that I know what I must do ... [pause] I thought of dividing 37 by 8, the men with the 8 hours and 23 by 8, the women with the 8 hours.

I : Why did you think to do these divisions?

N: [no response]

I : What did you find from these divisions Niki?

N: 4 and 2.

I : Can you verify your answers?

N: The men will be working 4 hours and the women 2.

I : When will the men be working 4 hours and the women 2?

N: [She reads the problem's question] : How many hours of work complete together in a day, if everyone works eight hours per day.

Niki performed two divisions but could not explain why. When she attempted to justify the results from the divisions, she referred only to the quotients, ignoring the remainders. Despite the fact that Niki has acquired computational skills and most of the times performs calculations correctly, she seems to have difficulties in choosing the operation appropriate to the problem. In the pre-test she did not attempt to solve this problem as she had found it too difficult, but in the post-test she attempted a solution although unsuccessfully.

Niki in the fifth problem provided an irrelevant method of solution. When questioned about it, she explained: "I remembered another problem we had solved in class. That's why."

Niki's performance on both tests was very low and her behaviour in both interviews appeared unchanged. The teaching experiment-2 did not seem to have any influence on Niki's learning behaviour.

Argyris

Argyris in the first problem ringed the incorrect mathematical expression 312x12, while in the pre-test he had ringed two incorrect expressions 312x12 and 12+312. When I asked Argyris about his solution method the following interaction took place:

I : Argyris what was your reasoning in the first problem?

A: I shall do a multiplication.

I : What will you multiply?

A: 312 by 12.

I : Why will you multiply?

A: What do you mean why?

I : What made you think of multiplying ?

A: I shall do a multiplication ... [pause].

I : Yes, but can you explain why?

A: I forgot again.

Argyris in the second problem was also unsuccessful. He ringed the expression 22+27, while in the pre-test he had ringed 22-27. When I interviewed him about this problem the discussion went as follows:

I : Argyris, can you explain to me your thinking in the second problem?

A: I shall do an addition.

I : Why?

A: Because ... [pause].

I : How many km does he cover in one day?

A: 27

I : How shall we find how many km he covers in 22 days?

A: Addition.

I : What shall we add?

A: [He points to the expression 22+27.]

I : Can you tell me why?

A: [no response]

Argyris in the third problem ringed an incorrect mathematical expression 520+260 but he had difficulty in justifying it. In the pre-test he had ringed the expressions 520+260 and 260+520. When I questioned him about the third problem the following interaction ensued:

I : Can you explain to me how you solved the third problem?

A: Addition.

I : What did you add?

A: 520 plus 260.

I : Why did you add 520+260?

A: Because ... [pause] because ... [pause].

I : What made you think that you should add?

A: [no response]

Argyris had left the last two problems (4 and 5) but during the interview I asked him about them:

I : Argyris, did you try the last two problems?

A: [no response]

I : Do you find difficulty with the problems?

A: Very much.

I : Where do you find the difficulty?

A: I don't know what make it difficult for me, but I find difficulty.

I : Do you understand the text?

A: Yes, I understand it.

I : But, how do this happen?

A: I don't understand what operation to use. [He points at the mathematical expressions in the first three problems and comments] : I can do these, but I cannot find them.

I : How did you find the operation in the first three problems?

A: I read the problem, looked at the operations and ringed the correct one.

I : Did you think about the operation beforehand or not?

A: No, I looked at the operations and I understood which was the correct one.

In the first three problems, in the pre-test, Argyris ringed incorrect mathematical expressions but did not make an effort in the last two problems. During the pre-interview, when I asked him about the first three problems he offered explanations, such as "I am not good in math.", "Others have a 'better' mind.", "Others are more clever than me."

However, Argyris' performance in the post-test indicated that, in a sense, he was not completely wrong as the second problem could be solved by repeated addition while the third could be solved by complementary addition rather than subtraction.

The students' overall performance

Maria's performance in both tests was excellent. However the explanations given in the post-interview seemed improved from the explanations given in the pre-interview. This change in behaviour could be attributed to two reasons. First that Maria knew what type of explanation was wanted and second that during collaborative group work she used to undertake the role of the tutor.

Takis', George's and Stella's performance at the post-test was better than the pre-test. One reason for this it could be that collaborative group work helped these students to change their attitudes towards word problem solving and this had an influence on their performance. Slavin (1992) argues that group member support for performance may enhance individual motivation and thus individual performance.

Another explanation for these students' improved performance at the post-test, could be that during the actual teaching stage, they worked on problems of a greater degree of difficulty. The students faced a variety of problems and therefore at the post-test the problems 4 and 5 were relatively simple. On the other hand, Niki and Argyris showed no improvement.

4.5.2 Changes in students' attitude

In order to examine any changes in the students' attitudes towards mathematics as a result of the actual teaching stage in TE-2, the attitudes' scales questionnaire was administered to them. The questionnaire was given to students immediately after the post-test, and it was identical to the questionnaire they had completed after the pre-test. All students completed the questionnaire without any procedural difficulty. During the interview on the post-test, the students were also asked to justify their responses on the questionnaire.

Below, the students' responses and results on each section of both questionnaires are compared with attempts to analyse the results.

Effort

The results of the 'effort' section on the attitudes' questionnaire (pre and post) as well as the responses of the individual students' effort on mathematics (before and after group work) are shown in the Table 4.2 and Table 4.3 respectively.

Table 4.2 The results of the 'effort' section on the attitudes' questionnaire

(The stem for all items is "I feel pleased in math when ...")					
EFFORT		**YE**	**yes**	**no**	**NO**
A1 The problems make me think hard.	**Pre**	1	5		
	Po	3	2		1
A2 What the teacher says makes me think.	**Pre**	3	2	1	
	Po	4	2		
A3 I keep busy.	**Pre**		2	2	2
	Po	3	1		2
A4 I work hard all the time.	**Pre**	3	2	1	
	Po	3	2		1

Table 4.3 Students' effort on mathematics before and after group work

	Maria		**Takis**		**Stella**		**George**		**Niki**		**Argyris**	
	pre	**po**	**pre**	**po**	**pre**	**po**	**pre**	**po**	**pre**	**po**	**pre**	**po**
Effort												
A1	yes	yes	yes	NO	yes	YE	yes	YE	yes	yes	YE	YE
A2	YE	yes	yes	YE	YE	YE	no	yes	YE	YE	yes	YE
A3	no	YE	no	NO	yes	YE	yes	yes	NO	NO	NO	YE
A4	YE	YE	YE	NO	yes	YE	no	yes	YE	yes	yes	YE

The results suggest that before and after collaborative group work, all students felt motivated to work hard. However, after group work the students felt even more motivated to do so.

In the pre-questionnaire George was the only student who had two 'no' responses on this group of items (i.e. A2 and A4). When he was asked about his response on the item "I feel pleased in math when what the teacher says makes me think" (A2) he replied: "I put 'no' because I know most of what the teacher says." His second 'no' response on the item "I feel pleased in math when I work hard all the time" (A4), it seems to reflect his lack of attention to the lesson. George rarely concentrates on the lesson and always finds something to distract him. He is a very active student and often needs the teacher's intervention to bring him back to the lesson. However he occasionally follows with the rest of the class.

The results on this group of items in the post-questionnaire indicate that all students except Takis felt motivated to put effort into their work. Takis, although a high-attainer, felt less motivated to work hard. In three (A1, A2, A4) of the four items on this group, he strongly disagreed ('NO'). These three items together with Takis' explanations are listed below.

On the item "I feel pleased in math when the problems make me think hard" (A1), he replied: "No, because then I feel sad and tired." On the item "I feel pleased in math when I keep busy" (A3), he responded: "No, because then I feel so tired." On the item "I feel pleased in math when I work hard all the time" (A4), his comment was simply : "No."

Takis' responses on the items A1, A3, A4 seem to reflect his attitude towards putting effort into his work. Finishing the problem as quickly as possible appeared to be a priority for him. As a consequence, he may omit solution steps or make careless mistakes.

In the pre-test, in the third problem, he ringed the correct mathematical expression but inversely (i.e. 260-520). When he started his explanation he immediately realised his error and corrected it. When he was questioned about this he explained: "I didn't notice that, I was in a hurry." In the fourth problem he found how many hours the men work in a day as well as how many the women work in a day, but he did not perform the addition to determine how many hours they both work in a day. When he was asked about this he said: "Oh! Oh! I forgot to do an addition ... I read the problem quickly." However Takis did not repeat these mistakes in the post-test. Niki was also another student who strongly disagreed with the item A3. But when she was questioned about her response, she did not offer an explanation.

Understand and Collaborate

The results of the 'understand and collaborate' section on the attitudes' questionnaire (pre and post) in addition to the responses of the individual students' attitudes towards understanding and collaborating in mathematics (before and after group work) are shown in the Table 4.5 and Table 4.6 respectively.

Table 4.5 The results of the 'understand and collaborate' section on the attitudes' questionnaire

(The stem for all items is "I feel pleased in math when ...")					
UNDERSTAND AND COLLABORATE		**YE**	**yes**	**no**	**NO**
B1 Something I learn makes me want to	**Pre**	4	2		
find out more.	**Po**	3	3		
B2 Everyone understands the work.	**Pre**		3	1	2
	Po	2	3	1	
B3 We help each other figure things out.	**Pre**		1	3	2
	Po	3	3		
B4 We understand each other's ideas about	**Pre**		1	4	1
math.	**Po**	2	3		1
B5 We explain our ideas to other students.	**Pre**			3	3
	Po	1	4	1	
B6 We understand instead of just getting	**Pre**	3	2	1	
answers to problems.	**Po**	2	4		
B7 We don't give up on really hard	**Pre**	1	2	2	1
problems.	**Po**	2	2	1	1

Table 4.6 Students' attitudes towards understanding and collaborating in mathematics, before and after group work

	Maria		**Takis**		**Stella**		**George**		**Niki**		**Argyris**	
	pre	**po**	**pre**	**po**	**pre**	**po**	**pre**	**po**	**pre**	**po**	**pre**	**po**
Un-Col												
B1	yes	YE	YE	yes	YE	YE	yes	yes	YE	yes	YE	YE
B2	yes	yes	yes	YE	yes	no	no	yes	NO	yes	NO	YE
B3	no	YE	no	yes	no	yes	yes	yes	NO	YE	NO	YE
B4	no	yes	no	NO	no	yes	no	YE	yes	yes	NO	YE
B5	NO	yes	no	yes	NO	no	no	yes	no	YE	NO	yes
B6	yes	yes	YE	yes	yes	YE	no	yes	YE	yes	YE	YE
B7	yes	NO	YE	yes	yes	YE	no	yes	NO	no	no	YE

The students' responses in this group of items on the pre- and post-questionnaire present a different picture. By studying the results it could be argued that before collaborative group work, the students tended in some way to be motivated towards understanding but not towards collaborating. It should be made clear here that if students did not take into account the stem ("I feel pleased in math when ...") then their responses would be more likely to represent facts concerning their mathematics classroom teaching rather than their feelings about mathematics. However, after collaborative group work the students felt motivated towards both understanding and collaborating.

In the pre-questionnaire they all agreed that they were interested in finding out more about mathematics (item B1), even in Argyris case. The students who responded positively to the item "I feel pleased in math when everyone understands the work" (B2) were the high-attainers and medium-attainers in class (Maria, Stella, Takis), while the low-attainers disagreed strongly.

The results suggest that there was no collaboration among students (B3). The only one who responded positively ('yes') on the item "I feel pleased in math when we help each other to figure things out" was George, who was sitting next to Argyris. When George was asked about his response he gave the explanation: "I help my classmate Argyris. He has a lot of difficulties. That's why."

The responses on the items "I feel pleased in math when we understand each other's ideas about math" (B4) and "I feel pleased in math when we explain our ideas to other students" (B5) showed that students felt more motivated not to explain their ideas to the others and furthermore they felt more motivated not to understand each other's ideas about math. One explanation for this, which comes from the preliminary observations, is that students were not provided time to explain their thoughts, and their participation in the lesson was limited to single words or numbers. It could be argued that their experiences in class were reflected in their responses to these items on the questionnaire.

The students' positive responses to understanding rather than finding answers to problems (item B6) can be explained by the high conformity to the teacher's solution method. It is worth mentioning that the students did not even have the opportunity in class to pick up at random the numbers in a problem and work out a solution. The solution approach was outlined by the teacher and the students simply had to carry it out. They were always working under the teacher's close supervision. However, it was not clear what the students really meant by 'understanding'. Did they mean knowing why they use a particular operation, or did they mean simply getting the right answer?

In the last item of this section of the questionnaire, the students who tended not to give up on really difficult problems (B7) were the high-attainers in the class, contrasted with the low-attainers who appeared to give up in difficult problems.

On the other hand, the students' responses in the post-questionnaire, indicate that all of them, as in the pre-questionnaire, felt motivated to learn more about mathematics (B1). The only student who disagreed with the item "I feel pleased in maths when everyone understands the work" (B2), was Stella. When she was questioned about her 'no' response, she explained: "I don't like it. It's as though ... [pause] I feel as if somebody is cheating. I want to think by myself, to try alone". Stella's response indicates that she felt that she learned better independently or that she was more concerned about herself than the others. This could explain her behaviour during the first week of collaborative group work when she was hesitant to show her work to the other group members.

Takis who agreed with the item B2 offered a characteristic explanation: "You know. I have feelings. If somebody in class gets a D and starts crying, what do you want me to do? Am I responsible too?" Takis' response implies that he felt motivated towards the item "Everyone understands the work" because he would not like to find himself in such a dilemma.

The students responses on the item "I feel pleased in math when we help each other to figure things out" (B3), indicate that all of them felt motivated towards collaborative group work. Furthermore the students who felt more motivated were the low-attainers (Argyris, Niki), and the high-attainer (Maria) students in grade-four class. It could be also argued that the collaborative group work in which the students were engaged in, had some influence on their responses to this item of the questionnaire.

The students felt motivated towards an understanding of each other's ideas about maths (B4) except Takis who disagreed strongly ('NO'). When he was asked about his 'NO' response on this item, he explained: "No, because some of our thoughts are not correct and if somebody understands and follows them ... [pause and gesture] we get 100 times the punishment." Takis' response revealed a teacher's action which never occurred during the preliminary observations.

During the interview about the post-questionnaire, the students appeared concerned firstly, about their performance in a test and secondly, about other students' comments in class. When George was asked to explain his 'YES' response on the item B4 he stated: "It's good to understand the others' thinking. Because if you work together and you don't understand the solution and you are asked to come up to the board, then you may do it wrong. And then the others will ask you why. But if you had understood the others' thinking, you would do it right."

The students also appeared pleased in math when they explained their ideas to other students (B5). However Stella disagreed. When she was questioned about her disagreement, she expressed some concerns about writing a test and working individually. The following interaction reveals Stella's feelings on this item:

I : [I read the item B5.] Stella you ringed 'no'. Can you explain why?

S: I don't like it. Because if the others do not know the solution ... [pause] and if it is a test ... [pause] and the teacher goes out of the classroom and somebody who doesn't know something asks and gets the solution ... [pause] I don't like this.

I : Okay. When you have a test I understand, but how do you feel in class about this?

S: Sometimes I don't like it.

I : Why?

S: Sometimes when I know something, and I have written it down, the others copy off me, like Niki, who always does that. And I don't like this. They should think by themselves.

It should be pointed out that during the preliminary sessions, none of the boys was observed copying from Stella, as they were sitting on the second row which was at some distance from the girls' row. Neither was Maria observed copying Stella's work. The only student who could see her work was Niki, who was sitting close to her. However, Stella never complained about Niki's behaviour in class. It may be the case that Stella uses the impersonal expression "others cheat on me", on purpose in order to justify her only example, who is Niki.

All students felt motivated towards understanding instead of producing answers to problems (B6). Maria on this item explained: "If you do not understand the problems and you only look for the answers, then you may make a mistake and then you will have to do double work."

On the last item of this group, "I feel pleased in math when we don't give up on really hard problems" (B7), Maria and Niki responded negatively. Maria's response is worth mentioning: "No, because if the problem is very difficult and we spend a long time and we do not know how to solve it, it's not right. We are wasting time." Maria's phrase "We are wasting time" justifies her positive responses to the first group of items (Effort), establishing her as a hard working student in the grade-four class. George justified his positive response ('yes') on this item (B7) with the following reason: "I don't give up on hard problems. Because if we are doing a test and I skip a hard problem, then I cannot get a mark for it." Again, the concern about the performance on a test, appeared to be a priority for these grade-four students.

Ego

The results of the 'ego' section on the attitudes' questionnaire (pre and post) and the individual students' attitudes towards being superior to their classmates (before and after group work) are shown in the Table 4.6 and Table 4.7 respectively.

Table 4.6 The results of the 'ego' section on the attitudes' questionnaire

(The stem for all items is "I feel pleased in math when ...")						
EGO		YE	yes	no	NO	y-n
C1 I know more than others.	**Pre**	1	1	1	2	1
	Po	2	3	1		
C2 I finish before my friends.	**Pre**		1		2	3
	Po	1	3	1	1	
C3 I get more answers right than my friends.	**Pre**		2	1	2	1
	Po	3	3			
C4 I am the only one who can answer a question.	**Pre**	1		3	2	
	Po	2	2	2		

Table 4.7 Students' attitudes towards being superior to their classmates, before and after group work

	Maria		Takis		Stella		George		Niki		Argyris	
	pre	po	pre	po	pre	po	pre	po	pre	po	pre	po
Ego												
C1	no	yes	YE	YE	y-n	YE	yes	yes	NO	yes	NO	no
C2	y-n	yes	yes	yes	y-n	YE	y-n	yes	NO	no	NO	NO
C3	y-n	yes	yes	YE	no	YE	yes	yes	NO	yes	NO	YE
C4	NO	yes	no	YE	YE	YE	no	yes	NO	no	no	no

On the ego group of items, in the pre-questionnaire, students encountered a procedural difficulty. They asked for a 'yes-no' point scale although there was no such provision. To overcome this difficulty the students were asked to think really hard and ring the response that seemed to reflect their true feelings. However three students (Maria, Stella, George), in some items, ringed both responses i.e. 'yes' and 'no'. The results showed that they had mixed tendencies towards their ego orientation. The low-attainers, Niki and Argyris, felt the least motivated in being superior to their peers. If students treated this group of items as independent, without reading the stem at the beginning of each item, then their responses represent classroom facts rather than reflections of opinions.

In the post-questionnaire, the results suggested that most students felt motivated to be superior to their peers. Those who expressed the opposite feelings were again the low-attainers Niki and Argyris.

On the first item of this group, "I feel pleased in math when I know more than others" (C1), Argyris disagreed. When he was questioned about this he explained: "No, because I want the others to be like me. I do not want to be the only one." The students who agreed with this item expressed reasons concerning the teacher's feelings, their performance in a test, and their ability in mathematics.

Takis explained: "Yes, I feel very pleased because I know that I will get a good mark and my teacher will be pleased." George stated: "When I know more than the others in the tests or in the questions, I am happy because I did them correctly." Stella said: "I like to be clever." Takis and George in their responses referred, although indirectly, to performance in a test.

On the next item "I feel pleased in math when I finish before my friends" (C2), those who responded negatively were the low-attainers, Niki and Argyris. When they were questioned about their responses, Niki commented: "It is not nice. I do not like it.", and Argyris explained: "My classmates play a game which says: 'The first the worst, second the best, third with the hairy chest'. That's why. I like to stay behind with my friends. I don't like to be the first."

Despite Argyris' explanation, during the preliminary sessions, it was noticed that most of the students, except him, were competing for who would finish first. In fact there were times that Takis (mainly) and George finished the activities first.

Takis' positive response ('yes') on this item was also characteristic: "Yes, I like it. Because I get rid of anxiety." Takis' feeling towards finishing the activities before his classmates, could be explained by his earlier stated motivation to avoid working hard.

All students appeared to feel pleased in math when they have more right answers than their friends (C3). George comparing himself with the other students in class stated: "My classmates always get more answers right than me. So when this happens with me, I feel happy."

On the item "I feel pleased in math when I am the only one who can answer a question" (C4), the low-attainers, Niki and Argyris were the only students who felt less motivated to be superior to their classmates. Niki, when was questioned about this item, explained: I don't feel pleased. It's not nice if I am the only one who can answer a question. I feel left out."

Maria who responded positively to this item commented: "Yes, because sometimes it happens like this ... [pause] Then the teacher thinks that I am clever."

Conformity

The results of the 'conformity' section on the attitudes' questionnaire (pre and post) as well as the individual students' conformity to the solution methods of the teacher (before and after group work) are shown in the Table 4.8 and Table 4.9 respectively.

Table 4.8 The results of the 'conformity' section on the attitudes' questionnaire

(The stem for all items is "I feel pleased in math when ...")					
CONFORMITY		**YE**	**yes**	**no**	**NO**
D1 We solve the problems the way the teacher shows us and don't think up our own.	**Pre**	5	1		
	Po	3	2	1	
D2 We all solve the problems the same way and don't think up different ways.	**Pre**	2	3	1	
	Po	2	1	3	

Table 4.9 Students' conformity to the solution method of the teacher, before and after group work

	Maria		**Takis**		**Stella**		**George**		**Niki**		**Argyris**	
	pre	**po**	**pr**	**po**	**pr**	**po**	**pr**	**po**	**pr**	**po**	**pr**	**po**
Confo												
D1	yes	yes	YE	no	YE	YE	YE	YE	YE	yes	YE	YE
D2	yes	no	no	no	yes	YE	YE	yes	yes	no	YE	YE

The results suggest that before and after collaborative group work, most students felt pleased in math when they conform to the teacher's solution method (D1). Further, in the pre-questionnaire most of them felt pleased in math when they solve the problem as their classmates do (D2). In the post-questionnaire half of them felt pleased in math when they do not solve the problem as their classmates do, but when they think up different ways (D2).

In the pre-questionnaire the only differentiation to the conform items was Takis' 'no' response on the item "I feel pleased in math when we all solve the problems the same way and don't think up different ways" (D2). When asked about this, he explained: "As I told you before, there other students who think of 100 different ways to solve the problem but they are all wrong." However Takis explanation indirectly implies that all students solve the problems the same way. George's explanation about his 'YES' response on the item "I feel pleased in math when we solve the problems the way the teacher shows us and don't think up our own" (D1), was also characteristic: "I don't think up a different way because it would be wrong."

One explanation for the students' conformity to the solution method of the teacher, in the pre-questionnaire, is that the teacher was the only source of help in this grade-four class. Not only cooperation between students was non-existent but not even any time was allocated to students to explain their thoughts.

However, in the post-questionnaire, the results indicate that some of the students responded differently (i.e. D1: Takis, D2: Maria, Niki) . On the item "I feel pleased in math when we solve the problems the way the teacher shows us and don't think up our own" (D1), one student (Takis) disagreed. He justified his disagreement by saying that "No, because we may think of easier solution methods." Although during the preliminary sessions, Takis might have thought differently about a problem, he was never observed to offer an alternative solution method. Takis and the rest of the students appeared to follow the teacher's solution method. On the other hand, the students who responded positively to this item, expressed a variety of reasons. Below are the students' positive explanations:

M: Yes, because we do not have to think so much.

S: Yes, because if we follow a different way, it may be wrong.

G: Yes, because he shows us the correct way and thus we know what we must do and how to do it faster, and therefore we are not confused.

N: Yes.

A: Yes, because our own way may be wrong.

The last item "I feel pleased in math when we all solve the problems the same way and don't think up different ways" (D2), was the item with the maximum number of negative responses in this questionnaire. Half of the students felt pleased in math when they all solve the problems the same way and half of the students felt pleased in math when they think up different ways. The students who responded positively explained:

A: Yes, because if we follow a different way it may be wrong. And then we get punished.
S: Yes, because the other way may be wrong.
G: When I have followed a different way and the teacher asks me to come up to the board, my classmates will tease me because I thought differently.

The students who disagreed with this item offered the following explanations:

M: I ringed no, because if we think different ways, we learn more ways of solving the problem.
T: No, because we may think of easier ways.
N: No, because someone else may think of another way. But it has to be correct.

Marias', Takis', and Niki's "no" response on the item D2 shows the difficulty of interpreting questionnaires. Although all three students gave the same response ('no') on the item D2 they offered different explanations.

Argyris was the second student, after Takis, who made a reference to punishment. No other students referred to this. In fact this was an issue which emerged only from these two students in the interview procedure. George's positive response indicates that he avoids thinking differently because he is concerned with other students' comments. The two high-attainers, Maria and Takis, who disagreed on this item (D2), could probably have experiences of different solution methods. In fact, Maria in the pre-test and post-test solved the fourth problem differently from the rest of the students. Niki was the third student who disagreed, but her explanation implies that one can think differently but his/her solution method must be correct.

The results of the attitude questionnaire suggested that students before, and especially, after group work felt motivated to work hard. Before group work the students tended to be motivated towards understanding but not towards collaborating. However, after group work they felt motivated towards both understanding and working collaboratively. Although the students may have not read the stem in the pre-questionnaire, it could be also argued that group work, during the teaching experiment, caused some changes in the students' attitudes which were reflected on their responses on the post-questionnaire.

Furthermore, the results showed that the students had mixed tendencies towards their 'ego' orientation. Those who felt the least motivated to be superior to their peers appeared to be the low-attainers. Finally, the results suggested that before and after group work most students felt pleased in maths when they conformed to the teacher's solution method. However, before group work most of the students felt pleased when they solved the problems as their classmates, while after group work half of the students felt so.

Overall, the post-questionnaire showed a change in the students' attitude towards mathematics and particularly word problem solving. This change may be due to the intervening teaching experiment. Therefore, it could be argued that a change in teaching practice appeared to have positively influenced the students' attitude toward the subject.

In addition to the above results the questionnaire and the following interview revealed some interesting points:

- The difficulty of interpreting questionnaires (i.e. students gave the same responses but for very different reasons).
- Most of the information on students' attitudes comes from the interview procedure, and not the questionnaire.
- Students appeared reluctant to be critical, maybe because they were worried that I would report back to their teacher or that I would think badly of them.

CHAPTER 5 CONCLUSIONS: CHANGES IN ATTITUDES AND BEHAVIOUR WHILE MOVING FROM TRADITIONAL CLASS TEACHING TO GROUP WORK

Summary of conclusions: The strength of this thesis has been that it looks closely at the processes involved during the introduction of small group work. It particularly examined how the children challenged each other, how they interacted with one another and/or how they supported each other. The potential of achieving learning through social interaction seemed to be demonstrated. It also illustrated the difficulties that can arise.

On the basis of the preceding analysis it could be argued that teaching experiment-1 and teaching experiment-2 were successful in establishing a collaborative environment. In both experiments the students changed their attitudes towards group work and problem solving, and some changes regarding their learning behaviour were also observed. The style of teaching used in both experiments appeared crucial in facilitating classroom debate, both in group and whole class discussion. This gave the children a forum in which a wide range of solution methods could be expressed, thought over and modified. The conclusions synthesised from both experiments will be discussed in details below.

5.1 Almost all students were able to adapt to a group working style although those previously taught using a narrow and rigid strategy found it more difficult to adjust

During the preliminary observations, in both grade-four classes, the students appeared to be working under their teacher's close guidance. (Each class had a different teacher.) Problem solving was seen by the students as an activity which meant following the teacher's suggested steps. Opportunities to explain their thoughts, to suggest alternative solution methods, to take initiatives in the problem solving process, or to interact between them, were non-existent in these two grade-four classes (Section 3.1, 4.1).

Both teachers and particularly the principal (in teaching experiment-2), followed a transmission style of teaching (Schoenfeld, 1988, Cobb, 1988, Burkhardt, 1988), placing more emphasis on the ability to spot the operations and obtain the right answer rather than developing an understanding of the problem. Students were viewed as passive learners, who simply had to follow the teacher's instructions in order to form the operations, perform the calculations and complete the solution.

However, when the new problem solving approach, of the teaching experiments (1 and 2) was introduced in each of the two grade-four classes, the students' behaviour changed, although at differing rates in each class.

The students in teaching experiment-1 seemed to have more difficulties in meeting the expectations of the new approach than the students in teaching experiment-2. One explanation for this is that when the students of teaching experiment-1 appeared to be having difficulties with a problem, the teacher's typical behaviour was to provide them with key words. No other alternative strategy was used by the teacher. As a result, when the students were asked to follow the new approach, 'detaching' from this key word strategy to which they were so accustomed, they appeared disorientated and intimidated. However they managed to change their behaviour and to meet the expectations of the new approach.

The students' change of behaviour, in teaching experiment-1, went through the following phases:

At the beginning of the actual teaching stage, when the students started to work in groups, they appeared a little uncomfortable when asked to engage in problem solving activities with more focus on explaining and justifying their solution methods. Their responses revealed that they were accustomed to specifying the operation/s for the problems but unaccustomed to explaining their choices. Only four out of fifteen students were able to answer the 'why' question, while the rest were unable to. The students' refusal or inability to answer the 'why' question may be explained as a breakdown of the didactical contract (Brousseau, 1984) which had been established between them and their regular teacher (Section 3.4.2.1).

Half way through the actual teaching stage, as the students were working collaboratively and the 'why' question was being emphasised, it was noticed that their attitudes towards the solution of a problem had started to change. Their responses differed from those of the initial phase of the actual teaching stage. The students' responses now seemed to imply that in order to explain and justify a solution method it is important to have thought it through and understood the situation as described in the problem. At this point, the conflict of the didactical contract created at the beginning of the actual teaching had started to resolve itself.

Towards the end of the actual teaching stage the students' and the teacher's / researcher's behaviour had perceptibly changed. The students were coming up most of the time with explanations and justifications in terms of their problem solving strategies, and the teacher had changed his way of questioning students about their solutions. A 'new' relationship had been created between the teacher / researcher and the students and therefore a 'new' didactical contract had been established. The students, accepting their obligations stemming from this 'new' contract, were trying to satisfy them without any resentment at the change.

The students' behaviour in teaching experiment-1, suggests that it is sometimes difficult and time consuming to change classroom practices and behaviour which have been established through the power of habit. Students being accustomed to their teacher's practices, were unwilling, at the beginning of teaching experiment-1 to follow a new approach in problem solving. However as time passed, with the students being engaged in collaborative work and with the researcher's effort, their attitudes towards problem solving had undergone a change. Towards the end of teaching experiment-1 the students were able to satisfy the expectations of the new problem solving approach. Among the factors that appeared to contribute to the change of their behaviour were the researcher's patience and persistence. Both seemed necessary to facilitate the difference in the students' behaviour.

Unlike the students in teaching experiment-1, the students in teaching experiment-2 appeared to have fewer difficulties in accepting the new approach, although only one of the two groups actually conformed to the expected behaviour.

It could be argued that, although these students were taught by transmission style teaching, their teacher employed a variety of problem solving strategies in directing students to the appropriate operation and therefore the correct answer (i.e. providing hints, using cued elicitations, providing verbal explanations).

Since most of the students were familiar with using different approaches to reach the solution to a problem, they did not seem to be at a loss when asked to follow a new approach (i.e. explain and justify the solution method) and a new way of working (i.e. working in groups). Both groups seemed willing from the beginning of the actual teaching stage to follow the new approach and they did not object to the researcher's suggested guidelines.

5.2 Mixed ability groupings (of wide and/or narrow range) appeared to enhance the self-image and self-esteem of the relatively low attaining pupils within the group

In teaching experiment-1 most of the problems were solved collaboratively. In teaching experiment-2 all the problems were solved collaboratively. This was imperative in order to establish and maintain a high degree of collaborative group work.

Mixed ability groupings (of wide and/or narrow range) appeared to do much to enhance the self-image and self-esteem of the low (Costas and Dina) and medium (Stella) attaining pupils. However the research methodology was not designed to measure these phenomena.

Low attaining children who had previously found it difficult to participate in the classroom (Costas and Dina) seemed to gain more confidence in a group work situation (Section 3.4.2.3). The involvement and motivation levels of these two pupils increased noticeably. These findings are consistent with those of other researchers which have shown that cooperative efforts resulted in higher self-esteem than competitive or individualistic efforts (Johnson and Johnson, 1992).

Moreover a medium-attainer student (Stella) who showed limited participation in the group's discussion and refused to reveal her work to the other members, was encouraged by them. This resulted in an increase in her self-confidence and she slowly started to open up and gradually changed her behaviour.

Stella's building up of self-confidence appeared to be a result of both external and internal factors. Firstly, she was accepted by the group and secondly she could see that she could do the work and thus contribute to the group. Both factors seemed to help Stella to build her self-confidence and to contribute in a change in her attitude and behaviour towards group work in problem solving (Section 4.4.3.1).

5.3 A group of low and average attaining pupils was not effective in producing learning

The students during the first week of teaching experiment-2, were assigned in two mixed sex and mixed ability groups of narrow range (one group with high/ medium-attainers and one group with medium/low-attainers). They worked in these groups for five sessions. During this period it was observed that the two groups demonstrated different competencies regarding the solution of a problem.

The students in the first group were able to identify the operations for the problem, to justify them, to carry out the calculations, and finally to verify their answer. Most of the time, they had an outline of the solution. The students used this outline in order to arrive at the answer to the problem. Verifying the answer to a problem seemed to be a consequence of carrying out the operations which the students had previously identified and explained. The students in the first group appeared confident of their answer and they were able to verify it. In some cases the answer was double-checked.

When the students in this group met difficulties they were generally able to overcome them through discussion. They seemed to challenge, as well as support each other. They were rarely short of ideas about the solution or willingness for the work they had to do. They solved correctly 8 out of 9 problems. At the end of the teaching experiment all three students of group one improved their problem solving behaviour. This outcome is consistent with other research findings which have shown that in groups with high and medium-ability students, all students tended to be active participants (Webb, 1991).

The second group contained only two students for part of the time, although most of the time three students were present. In contrast to the first group, the students of the second group identified operations and performed calculations without justifying their reasoning (section 4.4.1.1). The choice of an operation was not followed by an explanation. An outline of the solution existed but always was rather incomplete. It could be used to begin the solution but was inadequate to complete it. Additional operations could be chosen during the process of problem solving but they were chosen on the basis of the reasonableness of the answer (section 4.4.1.2). Niki but chiefly George calculated the answer. In light of an unreasonable answer they used another operation. This is one of the strategies that students use in solving word problems (Sowder, 1989).

Further, the medium and low-attainers in this group, often found difficulties, could not progress, followed incorrect solution methods or even wanted to abandon the group work. Although there is research evidence that in such a group composition there arise 'questions eliciting help more frequently' (Webb, 1991, p. 379), it was noticed that in the above critical moments for group two, the students were unable to find assistance. There was no-one in the group capable of challenging George and Niki. As a result the students of the second group performed very poorly completing successfully only 2 problems out of 9. Therefore, the students of group two may have shown improvement in their cooperative skills; however, there was no significant change in their learning. Although this outcome does not conform to Webb's (1991) claim that mixed ability groups with medium and low-attainers promote active participation by most or all members, this finding has very limited value due to the fact that it reflects the behaviour of only one group which contained two members from a class with two groups.

5.4 Group work promoted interdependence

When George and Niki were in a separate group (group two), it was Niki who encouraged George to persist with the group work. She also supported him in the calculations. However, when they joined the other group (group one), it was noticed, that the behaviour of these two students was reversed. It now was George who encouraged and helped Niki (Section 4.4.2.3).

It could be argued that the positive interdependence developed between Niki and George, when they worked in a separate group, also followed them into the larger group. George seemed to feel responsible for Niki's productivity and felt obliged to assist her with the work. Niki's earlier efforts to convince George to continue with the work seemed to have been successful. Although other group members supported Niki too, it was George who was the one who did the most encouraging. This is in line with previous research findings that 'cooperative learning experiences promote greater interpersonal attraction and more positive relationships among students than do competitive and individualistic learning experiences' (Johnson and Johnson, 1985, p. 112).

5.5 Group work permitted the emergence of leaders with negotiating behaviour

During teaching experiment-2, Takis, but mainly Maria demonstrated leadership behaviour. These two students seemed to be leaders who operated through negotiation (Section 4.4.2.2).

Both students, Takis and Maria, stimulated negotiating behaviour in other group members. They were seen to initiate solutions and suggestions, raise questions and challenge statements and assertions in ways that encouraged the discussion to move forward, as described by Gold, 1998, Bennett and Dunne, 1992. They assessed the progress of the group and they sought to achieve agreement on discussions that were taking place.

5.6 Class discussion provided a forum for reflection and modification of students' solution methods

Classroom discussions were successful in providing an open forum for groups to express their solution methods and compare them with those of the other groups. Encouraging class discussion, after the students had solved a problem, enabled them to think, reflect, reorganise and evaluate their solution methods.

The data collected in the teaching experiment-1, indicated that during classroom discussions the students demonstrated the following behaviour (Section 3.4.1):

- When a group was asked to tell the class how they reasoned on the problem, all the group members made an effort to give the best explanation and justification (Johnson and Johnson, 1992).

- Group members presenting a solution method and finding it inappropriate, tended to reconsider and revise their thinking to find the correct method.

- The students hearing their class-mates justifying their solutions, tended to revise their thinking in the light of the correct answer. At other times thinking about other groups' solution methods and utilising these solutions in their own thinking helped the children decide on the appropriateness and correctness of these solutions.

- Group members explained their solution methods and provided arguments to justify them (Lampert, 1990). When they refuted an assertion they provided an explanation. Students meeting conflict points during the presentation of solution methods, sometimes resolved them through class discussion where the individuals expressed their opinions and tried to justify them (Perret-Clermont, 1980).

However, the above attitudes and behaviour were not demonstrated to the same degree by all groups or group members. There were some episodes where groups and/or particular group members had low participation.

5.7 Some but not all of the students with poor attainment and behaviour were integrated into group work

In establishing collaboration during teaching experiment-1, it was observed that there was a preliminary settling-in period, with increased unsettled behaviour. However, it felt important to see these periods through rather than be tempted to resort to professional exiting strategies. In the short term such strategies may well have seen an early return to more acceptable behaviour, but ultimately may have had a negative effect on establishing overall collaboration.

Several teaching dilemmas concerning mainly the case of two students Vasilis (in TE-1) and Argyris (in TE-2) were dealt with. Both of them at the beginning of the teaching experiment (1 and 2 respectively) showed extremely disruptive behaviour.

Vasilis had been rejected already by two groups and was unwilling to make an effort to integrate into a group. He did not seem to have a close friend in the class with whom he liked to collaborate. When he was asked to work together with the researcher, in order to let the groups work smoothly, he accepted. At the same time, the groups appeared 'relieved'. It was noticed that Vasilis did not ask to rejoin a group.

On the other hand, Argyris' poor behaviour in the first session of teaching experiment-2 created tension within the group. In the second session Argyris displayed extremely disruptive behaviour, became involved in incidents and was removed from his group. He worked separately for one and a half sessions. On a number of occasions Argyris was seen to be observing his group-mates involved in group work and it seemed as if he would like to rejoin his group. He appeared to feel left out. He did eventually take the initiative to rejoin his group at the beginning of the fourth session. When he was asked to leave the group, he burst into tears and promised to keep quiet. Argyris, indeed, kept his promise and he continued to work collaboratively for the remainder of the experiment. Although he had behavioural problems, he seemed to be able to exhibit acceptable behaviour.

Trying out new ways of working and re-sorting individuals to work with others with whom they do not choose to work with, may create real problems. These become more serious with students who have extremely disruptive behaviour or students who are far behind the others in knowledge and skills. As a result the groups tend to reject these students as they feel that they waste their time explaining to them how to behave or what tasks to do and how to do them. Vasilis and Argyris are examples of two such cases. Each of them with disciplinary and academic problems had been rejected by their groups. Vasilis made no efforts to follow any group, while Argyris took the initiative and rejoined his group. Argyris, seemed to like group work better and chose to remain joined by showing acceptable behaviour. Although, at the beginning, group work had made Argyris behaviour more disruptive, it was clear that, later, group work had helped him to solve his behaviour problem.

5.8 Group work seemed to be beneficial to medium and high-attainers in terms of their learning behaviour

During the teaching experiment-2, collaborative group work seemed to be beneficial to three of the six students in terms of their learning behaviour. One of the three students, George, was a medium-low attainer, and the other two, Takis and Maria, were high-attainers .

When George was in the same group with Niki and Argyris, he would identify the operations without justifying his reasoning, and look for the answer to the problem by examining the results of different operations (with different combinations of numbers). However, when all the students in the class worked in one group, George demonstrated improved learning behaviour. This though was not the case with Niki (Section 4.4.3.3).

When students discussed the solution of a problem, George appeared to participate actively. Every time he suggested an operation, he provided a reason for choosing it. This benefited George because now he was constructing meanings for the operations he used. The environment of the new group, enriched with more able students than George, challenging and supporting each other, appeared to have an influence on his behaviour. On the other hand, Niki's attitude remained the same. She seemed to work in the same way as previously. She performed operations without understanding why. The new group environment did not seem to have an influence on her, although some changes were expected. Therefore, it could be argued that collaborative group work had a positive outcome for George but not for Niki.

Collaborative group work seemed also beneficial for the high-attainers Maria and Takis. The collaborative environment appeared to enable them to engage in more elaborate explanations (Webb, 1991, Swing and Peterson, 1982, Peterson and Swing, 1985). Maria and Takis in their effort to communicate their understanding to the other group members tended to construct more elaborate explanations. Extending their own conceptualisation of the problem (Yackel et al., 1991) seemed necessary for constructing an explanation that would make sense to their group-mates (Section 4.4.3.2). Perhaps as a result of undertaking the role of the tutor, their understanding appeared to be strengthened.

In addition, every time Maria was explaining a solution method she used her own way of presenting the given and unknown information of the problem. By organising the information appropriately, using intonation or reformulating the problem she indirectly outlined the required operation (Section 4.5.1). Further on many occasions she thought of solutions and suggested approaches that the other students could not think of. Maria's unique way of reasoning probably was one of the reasons that she was an accepted leader within the group.

5.9 It seemed to be helpful to the establishment of successful group work to explain to the students how they had to work in groups and what sort of behaviour they were expected to demonstrate

The group skills necessary for addressing a shared problem solving activity (i.e. to ask, explain, justify) were explicitly taught to the students. Furthermore, modelling the desired behaviour to students seemed helpful for a smooth transition from individual work to collaborative group work.

Galton and Williamson (1992) stress the importance of teachers communicating to students how to collaborate. 'For successful collaboration to take place pupils need to be taught how to collaborate so that they have a clear idea of what is expected of them' (p. 43). They also explain that teachers 'should tell students what they are going to be doing and why - sharing with them the learning theory of working in groups' (Reid et al., 1982, cited in Galton and Williamson, 1992, p. 168).

The transcription of tapes from the teaching experiment-2 revealed the group skills that students used in the group's discussion. These group skills included activities such as the following:

- questioning / prompting (T: Yes, but can you explain why?);
- being able to organise themselves and keep the activity going (M: Let's do the first way first and then we talk about the second way.);
- suggesting solutions (G: We will add these two [520, 68] and then we will subtract them from 2.700.);
- supporting each other (N: Come on George ... Do the division. Come on. Do it.);

- explaining (S: Thus his son gets 3.500 drachmas per day.);
- guiding each other (N: Do you mean to divide?, N: That's better.); and
- commenting (S: That would be stupid. To pay 12.000 for 5 glasses.).

Such activities were observed in both groups and also in the large one where all the students worked together. However, they were not used to the same extent by all group members. More often they were used by Maria and Takis (high-attainers) and less often by Argyris (low-attainer).

Although we cannot be certain whether the students' behaviour was a result of the cooperative skills taught by the researcher, the fact that the students used the skills taught suggests that the training may have been beneficial.

Particularly group two were not producing their justifications for their choices of operations. One explanation for this might be that students misunderstood the activity and therefore were not clear what kind of behaviour was expected, or that they found the problems difficult and they could not work out the operations.

5.10 The teacher's role seemed important in managing the group work and influencing the nature of group and class discussion that took place

From the beginning of teaching experiment-1, it was made clear to the children what was expected of their groups and what they themselves were expected to do. The emphasis was on asking the students to explain and justify their solution methods. Providing the opportunities for children to switch from the conventional way of solving a problem ('what') to a more questioning way ('why'), encouraged students to think for themselves.

From the beginning of teaching experiment-2, group work was introduced. It was important to do this as the students were accustomed to individual work and were encountering group work for the first time. An idea developed in teaching experiment-1 concerned the use of a poster with the expected behaviour. In teaching experiment-2 this poster was modified. It seemed helpful to use two posters, one with (five) guidelines referring to the collaborative group work, and another with (two) guidelines referring to class discussion. It was noticed that these two posters seemed to affect the students' behaviour.

It was explained to the students how they had to work and what sort of behaviour they were expected to demonstrate during group work and class discussion. Time was spent modelling the desired behaviour. A collaborative learning activity on solving a word problem was performed through role play. It was made clear to the students that there would be no intervening in their group discussion.

High value was placed on encouraging the group members to talk together, listen to each other and work collaboratively. As mentioned above, when observing the groups it was made clear that there would be no interference in their conversations. This approach seemed to work over time, and the students eventually took ownership of their discussions. During group work and class discussion it was essential to be non-judgmental and to address necessary comments to the entire class rather than to individual students. An effort was made to minimise the risks to individual pupils self-confidence by refraining from evaluative feedback.

To assume collaborative techniques at the expense of other styles of teaching and grouping methods, whether this be friendship groupings or groups operating at similar levels, would be to misread this piece of work and take it out of context. What it being argued, is that teaching word problem solving through group work in two traditional classes in this school, this project as discussed was successful in establishing collaboration and enabling children to develop positive attitudes towards group work and problem solving. Furthermore some changes in their learning behaviour were observed. Peer interaction and peer support appeared to serve the basis for children's learning (Vygotsky, 1978, Bruner, 1986, Wood et. al., 1976, Yackel at al., 1991). However, this research project offers a way forward for those wishing to establish more collaborative group work in their classrooms.

5.11 Limitations of the research

According to the legislation of the Greek Ministry of Education primary classes may have a size of 15 to 30 pupils.

The fourth-grade class in TE-1 had a total of 15 students while in TE-2 it had only 6 students. Thus the experimental class in TE-1 could be considered a representative of the Greek classes in terms of its size, but this was not the case for the experimental class in TE-2.

Also, a fourth-grade class with 4 groups in TE-1 and 2 groups in TE-2 imposed limitations to the generalisations of the conclusions due to the small number of pupils. Similar classes with more students, and a greater variety of teachers, would have allowed more experimental analysis and, therefore, would have presented a more generalised picture.

A second limitation to the experimental analysis concerns the absence of a comparison group. This was basically due to pragmatic reasons. The Greek school in which I conducted my research had a total of six grades and only one class in each grade. So there was only one grade-four class in this school, and, therefore, carrying out the teaching experiment without a comparison group was inevitable. The use of a comparison group would have allowed me to compare the attitude and behaviour of the control students with those of the experimental students and explore possible changes.

Furthermore, a third limitation concerns the value of the results due to the recency of the instruction. That is, some of the positive results in the study could be explained by the recency of the instruction and not because of cognitive growth (i.e. that students really understood the material).

Thus the small number of students, the absence of a comparison group and the recency of the instruction, impose crucial limitations to my research. I must therefore express some reservations with respect to the validity of the conclusions of the study.

5.12 Improving and extending the scope of the research in the future

However if the researcher were to undertake this type of research again, there would be things that he would do differently. He was limited in terms of available curriculum time on a daily and weekly basis. Also the fact that he was not a regular teacher in the school made it difficult for him. Therefore more curriculum time available could have allowed the researcher to conduct the research at his own pace. In addition it could probably have allowed him to extend the research to other topics of mathematics or areas of the curriculum.

Taping of group conversations in teaching experiment-1 would have proved useful in providing detailed information about individual group discussions. Having an independent observer making systematic observations on group behaviour, in both experiments, could have proved useful in providing information about the social context of the group which was not captured by the tape-recorder.

Extending the research method over a longer time period, and involving the teachers in parallel classes in the school (if there were any), would have been beneficial to provide a comparison and extend the sample. If this research could be expanded to include other schools more valid generalisations and/or a more quantitative analysis would have been possible.

A detailed investigation into the complexities of children's self-image and self-esteem would have been both relevant and extremely interesting.

5.13 The distinctive contribution of the research

Although there is a lot of research work (mainly from the US) on large scale studies on group work, there is little research attention on small scale studies on group work, and in particularly on managing the change for a traditional classroom. From this point of view, my research is a small scale study that it looks in details at the processes involved during the introduction of small group work.

It particularly provided a detailed account of two traditional teaching style classrooms where interventions were introduced concerned with collaborative group work in the context of word problem solving. It demonstrated that change was possible; I have attempted to record my accounts of change, in particularly with regard to students' attitudes and behaviour. As a result of the study, it could be argued that, in both cases (TE-1 and TE-2) some changes regarding their learning behaviour were observed and the students changed their attitudes towards group work and problem solving.

Another outcome of my involvement was the shift of emphasis and changes in my own practice. Using aspects of an action research framework, I learned to change the form of my classroom questioning and to provide an ethos which encourage students to encompass large stretches of dialogue.

This was the result of changing the way I viewed classroom questioning. That is, understanding classroom questioning in terms of sharing knowledge, I realised more deeply how my questioning practice could create or deny opportunities for students to engage (actively) in the learning process.

Additionally, the approach I used did show that it is sometimes difficult and time consuming to change students' practices and behaviour which have been established by the classroom's regular teacher. It appeared that patience, persistence and effort on behalf of the teacher (myself) were necessary factors for bringing about change in the students' behaviour.

Finally, it appeared that my research was important for both students and teacher (myself). It engaged us in considering our own practices and in modifying our practices in the light of reflection and self-reflection. For students it seemed to provide an opportunity to think about their ways of approaching solution methods (through discussion and group decision) and for myself as a teacher / researcher, it provided an opportunity to reflect on learning through negotiation and also to reflect on action research as a way of improving teaching.

By looking in detail at two classroom interventions concerned with collaborative group work in the context of word problem solving, this study presents the students' change in their learning behaviour and also illustrates the difficulties which may arise from such a teaching strategy.

The unique contribution of this study is that it looks closely at the processes involved during a change from a very traditional class (teaching) to a group working scheme. The results provide limited evidence about how this can be done quite successfully, together with some insight into the classroom processes.

The key contribution of the study is the very detailed knowledge of the reporting of incidents in real classrooms with real students and the problems that can arise while implementing such a teaching strategy.

Overall, I believe the detailed description of the processes observed and the insights gathered make this case study research a useful piece of

work for other teachers / researchers wishing to establish collaborative work in their classroom and explore its possible effects.

BIBLIOGRAPHY

Adamopoulos, L. (1993a). The revised maths textbooks for grades four, five and six in the primary school. *Ta Ekpedeftika*, 31-32, 168-179.

Adamopoulos, L. (1993b). The revised maths textbooks for grades four, five and six in the primary school. *Euclid C*, 38, 93-102.

Adelman, C., Jenkins, D. and Kemmis, S. (1980). Rethinking case study: Notes from the second Cambridge conference. In Simons, H. (Ed.), *Towards a Science of the Singular: Essays About Case Study in Educational Research and Evaluation* (pp. 47-61). Norwich: Centre for Applied Research in Education, University of East Anglia.

Adger, C. T., Kalyanpur, M., Peterson, D. B. and Bridger, T. L. (1995). *Engaging Students: Thinking, Talking, Cooperating.* Thousand Oaks, California: Corwin Press Inc.

Alison, K. (1985). Action research: What is it and what can I do? In Burgess, R. G. (Ed.), *Issues in Educational Research: Qualitative Methods* (pp. 129-151). Philadelphia: Falmer Press.

Alexander, S. et al. (1988). Problem solving - Getting started. In Burkhardt, H., Groves, S., Schoenfeld, A. and Stacey, K. (Eds.), *Problem Solving - A World View* (Proceedings of the problem solving theme group, ICME 5) (pp. 51-52). Nottingham, England: Shell Centre.

Artzt, A. F. and Newman, C. M. (1990). *How to Use Cooperative Learning in the Mathematics Class.* Reston, VA: National Council of Teachers of Mathematics.

Artzt, A. F. (1996). Developing problem-solving behaviours by assessing communication in cooperative learning groups. In Elliott, P. C. (Ed.), *Communication in Mathematics, K-12 and Beyond*, (1996 Yearbook of the National Council of Teachers of Mathematics) (pp. 116-125). Reston, VA: National Council of Teachers of Mathematics.

Askew, M. and Wiliam, D. (1995). *Recent Research in Mathematics Education 5-16*. London: HMSO.

Askew, M., Brown, M., Rhodes, V., Johnson, D. and Wiliam, D. (1997). *Effective Teachers of Numeracy*. London: King's College.

Bandura, A. (1982). Self-efficasy mechanism in human agency. *American Psychologist*, 37, 122-147.

Barnes, D. and Todd, F. (1977). *Communication and Learning in Small Groups*. London: Routledge and Kegan Paul.

Basetas, C. (1995). *The Ability of Primary School Leavers in Solving Operations and Word Problems with Integer Numbers: An Empirical Research*. Athens: Bibliogonia.

Bassarear, T. and Davidson, N. (1992). The use of small group learning situations in mathematics instruction as a tool to develop thinking. In Davidson, N. and Worsham, T. (Eds.), *Enhancing Thinking Through Cooperative Learning* (p. 235-250). New York: Teachers College Press.

Bassey, M. (1999). *Case Study Research in Educational Settings*. MK: Open University Press.

Bauersfeld, H. (1988). Interaction, construction, and knowledge: Alternative perspectives for mathematics education. In Grouws, D. A. and Cooney, T. J. (Eds.), *Effective Mathematics Teaching* (pp. 27-46). Reston, VA: National Council of Teachers of Mathematics.

Beldin, R. M. (1993). *Teams Roles at Work.* Oxford: Butterworth-Heinemann.

Bennett, N. and Dunne, E. (1990). Implementing cooperative groupwork in classrooms. In Lee, V. (Ed.), *Children's Learning in School* (pp. 63-78). London: Hodder and Stoughton.

Bennett, N. and Dunne, E. (1992). *Managing Classroom Groups.* Cheltenham, England: Stanley Thornes.

Boaler, J. (1997). *Experiencing School Mathematics: Teaching Styles, Sex and Setting.* MK: Open University Press.

Booth, L. (1984). *Algebra: Children's Strategies and Errors.* Windsor, Berks: NFER-Nelson.

Brousseau, G. (1984). The crucial role of the didactical contract in the analysis and construction of situations in teaching and learning mathematics. In Steiner, H. G. (Ed.), *Theory of Mathematics Education: ICME 5 Topic Area and Miniconference* (pp. 110-119). Bielefeld, Germany: Institut fur Didaktik der Mathematik der Universitat Bielefeld.

Brown M., Hart, K. and Kuchemann, D. (1984). *Chelsea Diagnostic Mathematics Tests - Number Operations.* Windsor, Berks: NFER-Nelson.

Brown, M. (1985). Number operations. In Hart, K., Brown, M., Kerslake, D., Kuchemann, D. and Ruddock, G. (Eds.), *Chelsea Diagnostic Mathematics Tests - Teacher's Guide* (pp. 73-79). Windsor, Berks: NFER-Nelson.

Bruner, J. (1986). *Actual Minds, Possible Worlds.* Cambridge, MA: Harvard University Press.

Burkhardt, H. (1981). *The Real World and Mathematics.* Glasgow: Blackie.

Burkhardt, H. (1988). Teaching problem solving. In Burkhardt, H., Groves, S., Schoenfeld, A. and Stacey, K. (Eds.), *Problem Solving - A World View* (Proceedings of the problem solving theme group, ICME 5) (pp. 17-42). Nottingham, England: Shell Centre.

Burkhardt, H. and Schoenfeld, A. (1988). Problem solving - An overview. In Burkhardt, H., Groves, S., Schoenfeld, A. and Stacey, K. (Eds.), *Problem Solving - A World View* (Proceedings of the problem solving theme group, ICME 5) (pp. 3-5). Nottingham, England: Shell Centre.

Carr, W. and Kemmis, S. (1986). *Becoming Critical: Education, Knowledge and Action Research.* London: Falmer Press.

Charles, R. and Lester, F. (1984). *Teaching Problem Solving.* London, England: Edward Arnold.

Cobb, P. and Steffe, L. P. (1983). The constructivist researcher as teacher and model builder. *Journal for Research in Mathematics Education*, 14(2), 83-94.

Cobb, P. (1988). The tension between theories of learning and instruction in mathematics education. *Educational Psychologist*, 23(2), 87-103.

Cobb, P., Wood, T., Yackel, E., Nicholls, J., Wheatley, G. Trigatti, B. and Perlwitz, M. (1991). Assessment of a problem-centered second grade mathematics project. *Journal for Research in Mathematics Education*, 22(1), 3-29.

Cobb, P., Wood, T., Yackel, E. and Perlwitz, M. (1992). A follow-up assessment of a second-grade problem-centered mathematics project. *Educational Studies in Mathematics*, 23(5), 483-504.

Cohen, P.A., Kulik, J.A. and Kulik. C. - L. C. (1982). Educational outcomes of tutoring: A meta-analysis of finding. *American Educational Research Journal*, 191, 237-248.

D'Ambrosio, U. (1990). The role of mathematics education in building a democratic and just society. *For the Learning of Mathematics*, 10(3), 20-23.

Damon, W. and Phelps, E. (1989). Critical distinctions among three approaches to peer education. *International Journal of Education Research*, 13, 9-19.

Davis, E. J. and McKillip, W. D. (1980). Improving story-problem solving in elementary school mathematics. In Krulik, S. (Ed.), *Problem Solving in School Mathematics* (1980 Yearbook of the National Council of Teachers of Mathematics) (pp. 80-91). Reston, VA: National Council of Teachers of Mathematics.

Dunne, E. and Bennett, N. (1990). *Talking and Learning in Groups*. London: Macmillan.

Edwards, D. and Mercer, N. (1987). *Common Knowledge: The Development of Understanding in the Classroom*. London: Methuen.

Elliot, J. (1992). *Action Research For Educational Change*. MK: Open University Press.

Exarchakos, T. (1988). *Didactic of Mathamatics*. Athens: Ellinika Grammata.

Galton, M. and Williamson J. (1992). *Group Work in the Primary Classroom*. London: Routledge.

Gipps. C. (1992). *What We know About Effective Primary Teaching*. London: Institute of Education.

Glaser, B.G. (1978). *Theoretical Sensitivity: Advances in the Methodology of Grounded Theory*. University of California: Sociology Press.

Gold, A. (1998). *Head of Department: Principles in Practice*. London: Cassell.

Good, T. and Biddle, B. (1988). Research and the improvement of mathematics instruction: The need for observational resources. In Grouws, D. A. and Cooney, T. J. (Eds.), *Perspectives on Research on Effective Mathematics Teaching* (pp. 114-142). Hillsdale, NJ: Laurence Erlbaum.

Griffin, P. and Cole M. (1984). Current activity for the future: the zo-ped. In Rogoff, B. and Wertsch, J. V. (Eds.), *Children's Learning in the "Zone of Proximal Development"* (pp. 45-64). San Francisco: Jossey-Bass.

Hammersley, M. (1992). *What' s Wrong With Ethnography?* London: Routledge.

Hart, K. M. (1984). *Ratio: Children's Strategies and Errors.* Windsor, Berks: NFER-Nelson.

Hatzigeorgiou, A. (1990). Addition and subtraction with integers. *Euclid C,* 27, 8-24.

Henderson, K. B. and Pingry, R. E. (1953). Problem solving in mathematics. In Fehr, H. F. (Ed.), *The Learning of Mathematics: Its Theory and Practice* (Twenty-first Yearbook of the National Council of Teachers of Mathematics) (pp. 228-270). Washington, DC: The National Council of Teachers of Mathematics.

Hertz-Lazarowitz, R. (1992). Understanding interactive behaviors: Looking at six mirrors of the classroom. In Hertz-Lazarowitz, R. and Miller, N. (Eds.), *Interaction in Cooperative Groups* (pp. 71-101). Cambridge: Cambridge University Press.

Hitchcock, G. and Hughes, D. (1989). *Research and the Teacher.* London: Routledge.

Hoyles, C., Sutherland, R. and Healey, L. (1991). Children talking in computer environments: New insights into the role of discussion in mathematics learning. In Durkin, K. and Shire, B. (Eds.), *Language in Mathematical Education* (pp. 162-175). MK: Open University Press.

Johnson, D. C. (Ed.) (1989). *Children's Mathematical Frameworks 8-13: A Study of Classroom Teaching.* Windsor, Berks: NFER-Nelson.

Johnson, D. W. and Johnson, R. T. (1974). Instructional goal structure: Cooperative, competitive, or individualistic. *Review of Educational Research*, 44, 213-240.

Johnson, D. W. (1981). Student - student interaction: The neglected variable in education. *Educational Researcher*, 10, 5-10.

Johnson, D. W. and Johnson, R. T. (1985). Internal dynamics of cooperative learning groups. In Slavin, R., Sharan, S., Kagan, S., Hertz-Lazarowitz, R., Webb, C. and Schmuck, R. (Eds.), *Learning to Cooperate, Cooperating to Learn* (pp. 103-124). New York: Plenum.

Johnson, D. W. and Johnson, R. T. (1989). *Cooperation and Competition: Theory and Research.* Edina, MN: Interaction Book Co.

Johnson, D. W. and Johnson, R. T. (1992). Positive interdependence: Key to effective cooperation. In Hertz-Lazarowitz, R. and Miller, N. (Eds.), *Interaction in Cooperative Groups* (pp. 174-199). Cambridge: Cambridge University Press.

Johnson, D. W. and Johnson, R. T. (1994). Collaborative learning and argumentation. In Kutnick, P. and Rogers, C. (Eds.), *Groups in Schools* (pp. 66-86). London: Cassell.

Kothali, E. and Georgakakos, E. (1992). The problem and its solution: A teaching approach in the primary and the gymnasium. In Kalavasis, F. and Meimaris, M. (Eds.), *Issues in Teaching Mathematics* (pp. 281-309). Athens: Protasis.

Krulik, S. and Rudnick, J. (1988). Helping teachers become teachers of problem solving. In Burkhardt, H., Groves, S., Schoenfeld, A. and Stacey, K. (Eds.), *Problem Solving - A World View* (Proceedings of the problem solving theme group, ICME 5) (pp. 123-129). Nottingham, England: Shell Centre.

Krutetskii, V. A. (1976). *The Psychology of Mathematical Abilities in Schoolchildren* (Edited by Kilpatrick, J. and Wirszup, I.). Chicago: University of Chicago Press.

Kuhs, T. M. and Ball, D. L. (1986). *Approaches to Teaching Mathematics: Mapping the Domains of Knowledge, Skills and Dispositions.* East Lansing: Michigan State University, Center on Teacher Education.

Kutnick, P. (1988). *Relationships in the Primary Classroom.* London: Paul Chapman.

Kutnick, P. and Rogers, C. (1994). *Groups in Schools.* London: Cassell.

Laborde, C. (1990). Language and mathematics. In Nesher, P. and Kilpatrick, J. (Eds.), *Mathematics and Cognition: A Research Synthesis by the International Group for the Psychology of Mathematics Education* (pp. 53-69). Cambridge: Cambridge University press.

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal*, 27(1), 29-63.

Lave, J. (1992). Word problems: A microcosm of theories of learning. In Light, P. and Butterworth, G. (Eds.), *Context and Cognition* (pp. 74-92). London: Harvester Wheatsheaf.

LeBlank, J. F., Proudfit, L. and Putt, I. J. (1980). Teaching problem solving in the elementary school. In Krulik, S. (Ed.), *Problem Solving in School Mathematics* (1980 Yearbook of the National Council of Teachers of Mathematics) (pp. 104-116). Reston, VA: National Council of Teachers of Mathematics.

Lincoln, Y. S. and Guba, E. G. (1985). *Naturalistic Inquiry*. California: Sage Publications.

Matsagouras, E. (1995). *Collaborative Teaching*. Athens: Grigori.

Mavrogiorgos, G. (1985). Educational reformations, technocracy and technical control: The case of new books in the primary school. *Sichroni Ekpedefsi*, 21, 25-27.

McLeod, D. B. (1992). Research on affect in mathematics education: A reconceptualisation. In Grouws, D. A. (Ed.), *Handbook of Research on Mathematics Teaching and Learning: A Project of the National Council of Teachers of Mathematics* (pp. 575-596). New York: Macmillan.

Menchinskaya, N. A. (1969). Fifty years of Soviet instructional psychology. In Kilpatrick, J. and Wirszup, I. (Eds.), *Soviet Studies in the Psychology of Learning and Teaching Mathematics* (v. 1, pp. 5-27). Stanford, CA: School Mathematics Study Group.

Ministry of National Education and Religion (1987). *Curriculum of Primary School's Subjects*. Athens, OEDB.

Ministry of National Education and Religion (MNER) (1993a). *My Mathematics. Fourth Primary Grade*, v. 2. Athens: OEDB.

Ministry of National Education and Religion (1993b). *My Mathematics. Sixth Primary Grade*, v. 1. Athens: OEDB.

Ministry of National Education and Religion (1993c). *My Mathematics. Sixth Primary Grade*, v. 2. Athens: OEDB.

Ministry of National Education and Religion (1994). *Mathematics for the Sixth Primary Grade. Teacher's Guide.* Athens: OEDB.

National Council of Teachers of Mathematics (1989). *Curriculum and Evaluation Standards for School Mathematics*. Reston, Va: National Council of Teachers of Mathematics.

Nesher, P. and Teubal, E. (1975). Verbal cues as an interfering factor in verbal problem solving. *Educational Studies in Mathematics*, 6, 41-51.

Newell, A. and Simon, H. A. (1972). *Human Problem Solving.* Englewood Cliffs, NJ: Prentice - Hall Inc.

Nicholls, J. G., Cobb, P., Wood, T., Yackel, E., and Patashnick, M. (1990). Dimensions of success in mathematics: Individual and classroom differences. *Journal for Research in Mathematics Education*, 21, 109-122.

Noddings, N. (1985). Small groups as a setting for research on mathematical problem solving. In Silver, E. A. (Ed.), *Teaching and Learning Mathematical Problem Solving: Multiple Research Perspectives* (pp. 345-359). Hillsdale, NJ: Lawrence Erlbaum.

Osborne, J., Black, P., Boaler, J., Brown, M., Driver, R., Murray, R. and Simon, S. (1997). *Attitudes to Science, Mathematics and Technology: A Review of Research*. London: King's College.

Perret-Clermont, A. N. (1980). *Social Interaction and Cognitive Development in Children*. New York: Academic Press.

Peterson, P. L. and Swing, S. R. (1985). Students' cognitions as mediators of the effectiveness of small group learning. *Journal of Educational Psychology*, 77, 299-312.

Pirie, S. (1991). Peer discussion in the context of mathematical problem solving. In Durkin, K. and Shire, B. (Eds.), *Language in Mathematical Education* (pp. 143-161). MK: Open University Press.

Pollard, A. (1985). *The Social World of the Primary School*. London: Holt, Rinehart and Winston.

Pollard, A. (1997). *Reflective Teaching in the Primary School: A Handbook for the Classroom* (3rd ed.). London: Cassell.

Pottari, D. (1989). Learning difficulties in mathematics at the primary school. *Euclid C*, 21, 17-26.

Putnam, R. T., Lampert, M. and Peterson, P. L. (1990). Alternatives perspectives on knowing mathematics in elementaries schools. In Cazden, C. B. (Ed.), *Review of Research in Education*, Vol. 16 (pp. 57-150). Washington, DC: American Educational Research Association.

Rathmell, E. C. and Huinker, D. M. (1989). Using part-whole language to help children represent and solve word problems. In Trafton, P. R. (Ed.), *New Directions for Elementary School Mathematics* (1989 Yearbook of the National Council of Teachers of Mathematics) (pp. 99-110). Reston, VA: National Council of Teachers of Mathematics.

Reid, J., Forrestal, P. and Cook, J. (1982). *Small Group Work in the Classroom*, Language and Learning Project - Education Department, Western Australia.

Reyes, L. H. (1984). Affective variables and mathematics education. *The Elementary School Journal*, 84(5), 558-581.

Romberg, T. A. (1992). Perspectives on scholarship and research methods. In Grouws, D. A. (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 49-64). New York: Macmillan.

Schoenfeld, A. H. (1985). Metacognitive and epistemological issues in mathematical understanding. In Silver, E. A. (Ed.), *Teaching and Learning Mathematical Problem Solving: Multiple Research Perspectives* (pp. 361-379). Hillsdale, NJ: Lawrence Erlbaum.

Schoenfeld, A. (1988). Research on problem solving. In Burkhardt, H., Groves, S., Schoenfeld, A. and Stacey, K. (Eds.), *Problem Solving - A World View* (Proceedings of the problem solving theme group, ICME 5) (pp. 6-16). Nottingham, England: Shell Centre.

Schunk, D. H. (1984). Self-efficacy perspective on achievement behavior. *Educational Psychologist*, 19, 48-58.

Scieszka, J. and Smith, L. (1995). *Math Curse*. New York: Penguin.

Sharples, D. (1983). *An Overview of School Based Action Research.* Paper presented at Action Research in Classrooms and Schools Conference, Manchester Polytechnic, March.

Sharpley, A. M. and Sharpley, C. F. (1981). Peer tutoring: A review of the literature. *Collected Original Resources in Education*, 5, 7-C11.

Simons, H. (1980). *Towards a Science of the Singular: Essays About Case Study in Educational Research and Evaluation.* Norwich: Centre for Applied Research in Education, University of East Anglia.

Simons, H. (1996). The paradox of case study. *Cambridge Journal of Education*, 26(2), 225-240.

Skemp, R. R. (1989). *Mathematics in the Primary School.* London: Routledge.

Slavin, R. E. (1985). An introduction to cooperative learning research. In Slavin, R., Sharan, S., Kagan, S., Hertz-Lazarowitz, R., Webb, C. and Schmuck, R. (Eds.), *Learning to Cooperate, Cooperating to Learn* (pp. 5-15). New York: Plenum.

Slavin, R., Sharan, S., Kagan, S., Hertz-Lazarowitz, R., Webb, C. and Schmuck, R. (1985). *Learning to Cooperate, Cooperating to Learn.* New York: Plenum.

Slavin, R. E. (1987). Ability grouping and student achievement in elementary schools: A best-evidence synthesis. *Review of Educational Research*, 57, 293-336.

Slavin, R. E. (1989a). Cooperative learning and student achievement. In Slavin, R. E. (Ed.), *School and Classroom Organization*, (pp. 129-156). Hillsdale, NJ: Laurence Erlbaum.

Slavin, R. E. (1989b). *School and Classroom Organization.* Hillsdale, NJ: Laurence Erlbaum.

Slavin, R. E. (1992). When and why does cooperative learning increase achievement? Theoretical and empirical perspectives. In Hertz-Lazarowitz, R. and Miller, N. (Eds.), *Interaction in Cooperative Groups* (pp. 145-173). Cambridge: Cambridge University Press.

Sowder, L. (1989). Choosing operations in solving routine story problems. In Charles, R. and Silver, E. (Eds.), *The Teaching and Assessing of Mathematical Problem Solving* (2nd ed.) (pp. 148-158). Reston, VA: National Council of Teachers of Mathematics.

Spradley, J. P. (1980). *Participant Observation.* Orlando, FL: Holt, Rinehart and Winston, Inc.

Steffe, L. P. (1983). The teaching experiment methodology in a constructivist research program. In Zweng, M., Green, T., Kilpatrick, J., Pollak, H. and Suydam, M. (Eds.), *Proceedings of the Fourth International Congress on Mathematical Education* (pp. 469-471). Boston, Massachusetts: Birkhauser.

Swing, S. R. and Peterson, P. L. (1982). The relationship of students ability and small-group interaction to student achievement. *American Educational Research Journal*, 19, 259-274.

Thompson, A. G. and Thompson, P. W. (1989). Affect and problem solving in an elementary school mathematics classroom. In McLeod, D. B. and Adams, V. M. (Eds.), *Affect and Mathematical Problem Solving: A New Perspective* (pp. 162-176). New York: Springingler-Verlag.

Thompson, A. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In Grouws, D. A. (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 127-146). New York: Macmillan.

Topping, K. J. (1998). The effectiveness of peer tutoring in further and higher education: A typology and review of the literature. In Goodland, S. (Ed.), *Mentoring and Tutoring by Students* (pp. 49-69). London: Kogan Page.

Tresou - Milona E. (1991). Fixed, error solving strategies, that students follow in exercises of ordering decimal numbers. *Euclid C*, 29, 53-66.

Troulis, G. (1992). *Mathematics in the Primary School.* Athens: Grigori.

Verschaffel, L. and De Corte, E. (1997). Word problems: A vehicle for promoting authentic mathematical understanding and problem solving in the primary school? In Nunes, T. and Bryant, P. (Eds.), *Learning and Teaching Mathematics: An International Perspective* (pp. 69-98). East Sussex: Psychology Press.

Vygotsky, L. S. (1978). *Mind in Society: The Development of Higher Psychological Processes.* Cambridge, MA: Harvard University Press.

Webb, N. M. (1982). Student interaction and learning in small groups. *Review of Educational Research*, 52, 421-445.

Webb, N. M. (1989). Peer interaction and learning in small groups. *International Journal of Educational Research*, 13, 21-39.

Webb, N. M. (1991). Task-related verbal interaction and mathematics learning in small groups. *Journal for Research in Mathematics Education*, 22(5), 366-389.

Wolcott, H. F. (1981). Confessions of a trained observer. In Popkewitz, T. S. and Tabachnick, B. R. (Eds.), *The Study of Schooling: Field Based Methodologies in Educational Research and Evaluation* (pp. 247-263). New York: Praeger.

Wood, D., Bruner, J. S., and Ross, G. (1976). The role of tutoring in problem solving. *Journal of Child Psychology and Child Psychiatry*, 17, 89-109.

Yackel, E., Cobb, P., Wood, T., Wheatley, G. and Merkel, G. (1990). The importance of social interaction in children's construction of mathematical knowledge. In Cooney, T. J. (Ed.), *Teaching and Learning Mathematics in the 1990s* (1990 Yearbook of the National Council of Teachers of Mathematics) (pp. 12-21). Reston, VA: National Council of Teachers of Mathematics.

Yackel, E., Cobb, P. and Wood, T. (1991). Small group interactions as a source of learning opportunities in second-grade mathematics. *Journal for Research in Mathematics Education*, 22(5), 390-408.

APPENDICES

Appendix 1 Units taught during the TE-1

Unit title	Sessions
'Revision - Extension' (on inverse problems)	2
'How we solve problems of four operations'	2
'Problems with more unknowns'	3
'Problems on adding decimal numbers'	1.5
'Problems on subtracting decimal numbers'	1.5
'Problems of addition and subtraction of decimal numbers'	2
'Problems whose solution requires one operation'	1
'Solving problems with two ways'	1
'The same problem with inverse'	1
'Problems of integer and decimal numbers'	1

Units are taken from the textbook: *My Mathematics*, v. 2, fourth-grade

Appendix 2 Units taught during the TE-2

Unit title	Sessions
'How we solve problems of four operations'	4
'Problems with more unknowns'	1.5
'Solving problems with two ways'	2.5
'The same problem with inverse'	2

Units are taken from the textbook: *My Mathematics*, v. 2, fourth-grade

Appendix 3 Test given to students before the actual teaching stage in TE-1

Pre-test

Class: D

Name: ....

1. A basketball game was attended by 975 students. How many drachmas were collected if every student paid 450 drachmas per ticket ?

2. Visiting the vegetable market with my mother we bought 3 kilograms of apples at 245 drachmas per kilogram, and 4 kilograms of potatoes at 130 drachmas per kilogram. How many drachmas did we pay altogether ?

3. 14 primary school children went on a coach trip for the day. The cost of the coach trip was 9100 drachmas. How many drachmas did each child pay ? If each child was given 1000 drachmas for the trip, how many drachmas would they each have after paying for the coach trip ?

Appendix 4 Test given to students after the actual teaching stage in TE-1

Post-test

Class: D

Name: ....

Solve the following problems:

1. A technician earns 9.450 drachmas per day and his helper 7.245 drachmas. How many drachmas do they earn together per quarter of a year?
   (one month = 30 days)

2. A vendor collected 754.560 drachmas from selling peaches and 577.200 drachmas from apples. If the cost of the peaches was 240 drachmas per kilo and the cost of apples was 185 drachmas per kilo, how many kilos of each kind did he sell? Which kind did he sell more and how many?

3. In order to make a cake we used 1,650 Kgr of flour, 0,75 Kgr of sugar and 0,980 Kgr of other ingredients. When we baked the cake its weight was 2,4 Kgr. What was the cake' s weight before cooking it? Did it weigh more before or after cooking and by how much?

Appendix 5 Attitudes' questionnaire given to students before and after the actual teaching stage in TE-2

Attitudes' Scales Questionnaire				
Class: D				
Name:...				
I feel pleased in math when ...				
The problems make me think hard.	YES	yes	no	NO
I know more than others.	YES	yes	no	NO
We solve the problems the way the teacher shows	YES	yes	no	NO
us and don't think up our own.				
I get more answers right than my friends.	YES	yes	no	NO
We explain our ideas to other students.	YES	yes	no	NO
I keep busy.	YES	yes	no	NO
We don't give up on really hard problems.	YES	yes	no	NO
I work hard all the time.	YES	yes	no	NO
We understand each other's ideas about math.	YES	yes	no	NO
I am the only one who can answer a question.	YES	yes	no	NO
Something I learn makes me want to find out more.	YES	yes	no	NO
Everyone understands the work.	YES	yes	no	NO
We help each other figure things out.	YES	yes	no	NO
What the teacher says makes me think.	YES	yes	no	NO
We all solve the problems the same way and don't	YES	yes	no	NO
think up different ways.				
I finish before my friends.	YES	yes	no	NO
We understand instead of just getting answers to	YES	yes	no	NO
problems.				

Appendix 6 Students' performance in each problem of the pre-test and post-test in TE-2

Problem 1

Problem 1		
312 photographs must be placed in an album.	312x12	12+12
On each page of the album should be placed	12/312	312-12
12 photographs.	12+312	12x26
	12-312	312/12
How do you work out how many pages are		
needed?		

	Maria		Takis		Stella		George		Niki		Argyris	
	Pre	Po	Pre	Po	Pre	Po	Pre	Po	Pre	Po	Pre	Po
Problem 1	v	v	x	x	x	v	x	x	v	x	x	x

The first problem of the post-test, the division - 'grouping' - problem, was found to be the most difficult problem in the post-test. Only two students, Maria and Stella, ringed the correct mathematical expression (312/12).

Problem 2

Problem 2		
Everyday Pericles' father goes to work and	27x22	22-27
comes back with his car and covers 27 km.	22+27	27-22
	594/27	22/27
How many km does he cover in 22 days?	22x27	27+27

	Maria		Takis		Stella		George		Niki		Argyris	
	Pre	Po	Pre	Po	Pre	Po	Pre	Po	Pre	Po	Pre	Po
Problem 2	v	v	v	v	v	x***	v	x	v	v	x	x

*** When she started explaining her answer, she immediately realised her mistake and proceeded to give an explanation for the correct mathematical expression.

In the pre-test, the second problem was found to be the easiest for the students. All, except Argyris ringed the correct mathematical expression. However in the post-test only three students, Maria, Takis and Niki, ringed the correct expression.

Problem 3

Problem 3		
A truck driver has to drive 520 km to get from	260x2	520+260
Athens to Salonica. After driving 260 km he stops	260/520	520-260
for lunch.	520x260	520/260
	260-520	260+520
How do you work out how far he still has to drive?		

	Maria		Takis		Stella		George		Niki		Argyris	
	Pre	Po	Pre	Po	Pre	Po	Pre	Po	Pre	Po	Pre	Po
Problem 3	v	v	v	v	x	x	x	v	x	x	x	x

The third problem, was a subtraction problem, and was solved correctly in the post-test by three students (Maria, Takis and George).

Problem 4

Problem 4
37 men and 23 women are working in a factory. How many hours of work do
they complete together in a day (eight hours)?

	Maria		Takis		Stella		George		Niki		Argyris	
	Pre	Po	Pre	Po	Pre	Po	Pre	Po	Pre	Po	Pre	Po
Problem 4	v	v	v*	v	v*	v	v**	v	x	x	x	x

* They did not identify the last operation (addition).

** After the two multiplications he was engaged in a kind of erroneous results' analysis.

In the pre-test only Maria had correctly solved this problem. The other three students, Takis, Stella, George, solved part of the problem. They had identified and performed the first two multiplications but had failed to identify the addition for completing the solution.

However, in the post-test all four students successfully identified the required operations and performed the calculations correctly. Furthermore, at the post-interview they explained their solution method appropriately. The students seemed to have understood the situation described in the problem and knew how to apply these operations (multiplication, addition) to the given information.

The fourth and the fifth problem were multi-step problems. The students were expected to identify the operations as well as perform the calculations. The test's results indicate that all students, except Niki and Argyris performed well. It should be pointed out that during the teaching experiment-2, the students worked only on word problems of this format.

Problem 5

Problem 5
The mother bought 10 cans of milk at 100 drachmas per can, 4 kilos of sugar
at 150 drachmas per kilo and one kilo of cheese. She paid 3.600 drachmas
for all these.
How much did she pay for the cheese?

	Maria		**Takis**		**Stella**		**George**		**Niki**		**Argyris**	
	Pre	Po	Pre	Po	Pre	Po	Pre	Po	Pre	Po	Pre	Po
Problem 5	V	V	X	V	V	V	X	V	X	X	X	X

The students' performance on the fifth problem, which was a four-step problem, appeared to have improved. At the pre-test only Maria and Stella had managed to identify the appropriate operations and solve the problem, but Stella could not provide reasons for the operations she had used. In addition the other three